Bunch-Timing Measurement in the Muon Cooling Experiment Via $\mathrm{TE}_{0,1,n}$ RF Cavities

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K.T. McDonald and E.J. Prebys

Princeton U.

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Consider a $\mathrm{TE}_{0,1,n}$ cavity of length a, height a/α and peak field parameter

$$\eta = \frac{eE_0}{m\omega c} = 0.0223,$$

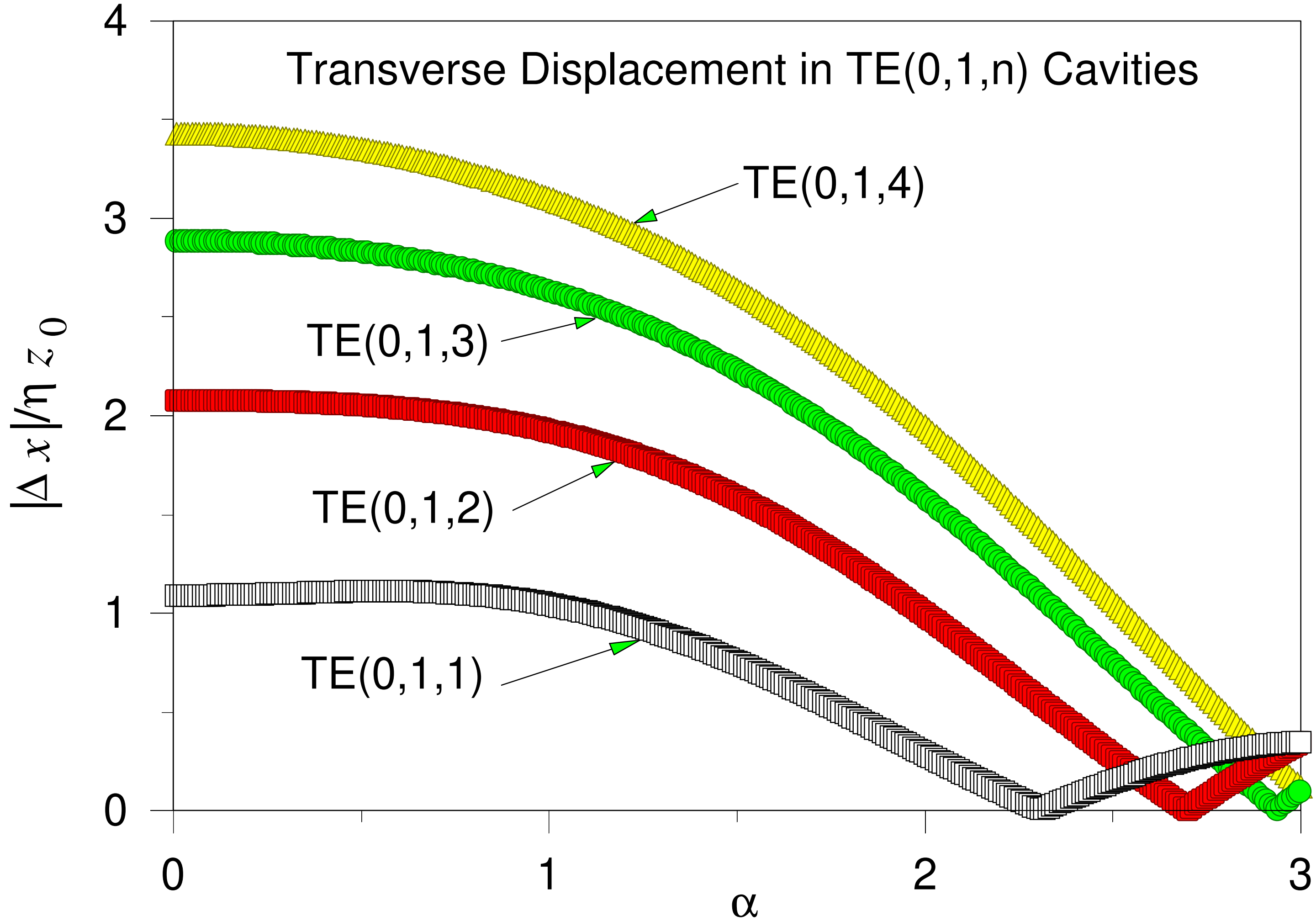
for $\omega = 2\pi 800$ MHz and $E_0 = 40$ MV/m.

$$\text{Dispersion relation: } \frac{\omega}{c} = \sqrt{n^2 + \alpha^2}\frac{\pi}{a}.$$

Transverse displacement across the cavity:

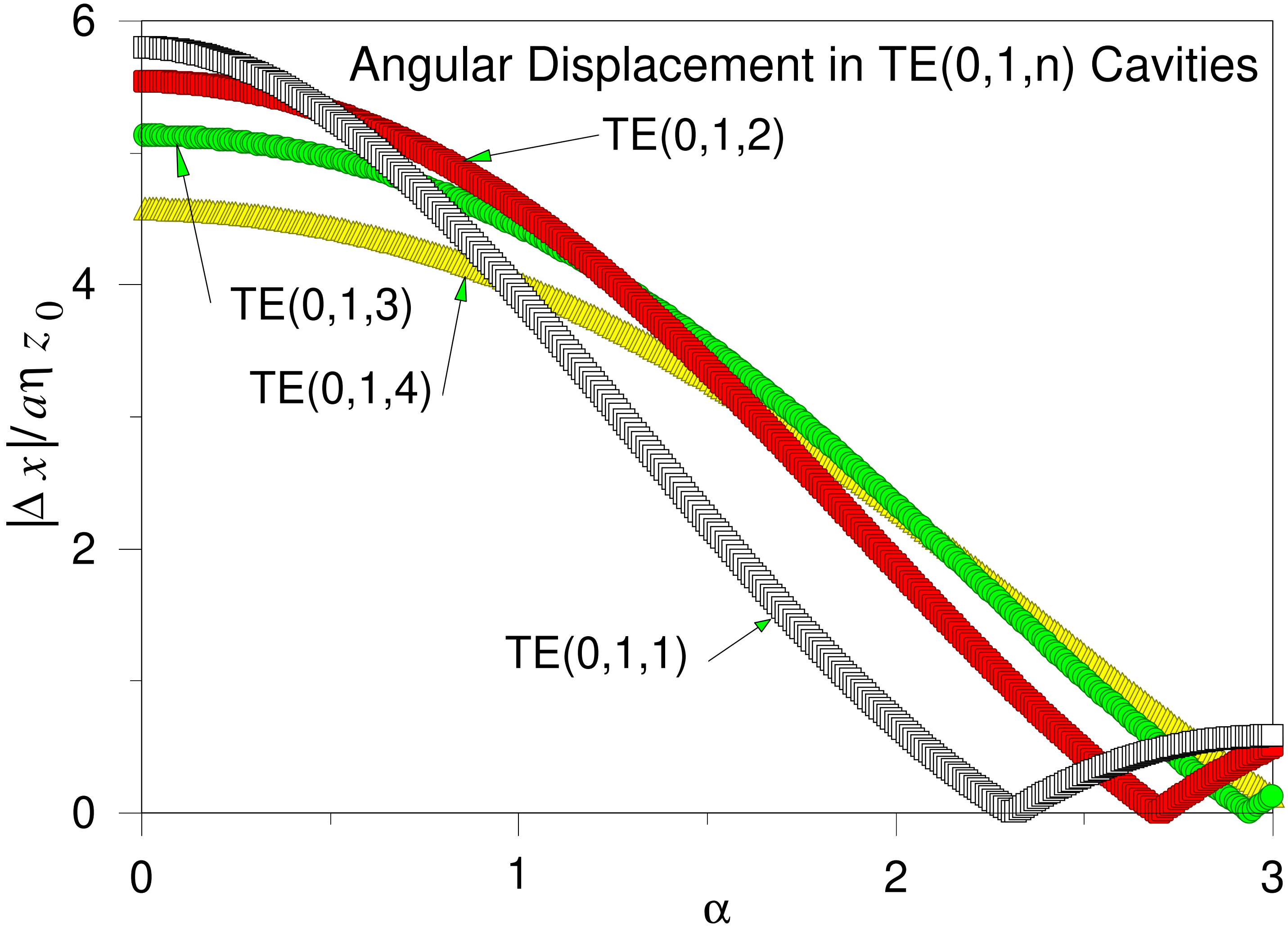
$$\frac{\Delta x}{\eta z_0} \approx \frac{2\gamma\sqrt{1+(\alpha/n)^2}}{1+(\gamma\alpha/n)^2} f\left(\frac{\sqrt{n^2+\alpha^2}\pi}{2\beta_z}\right),$$

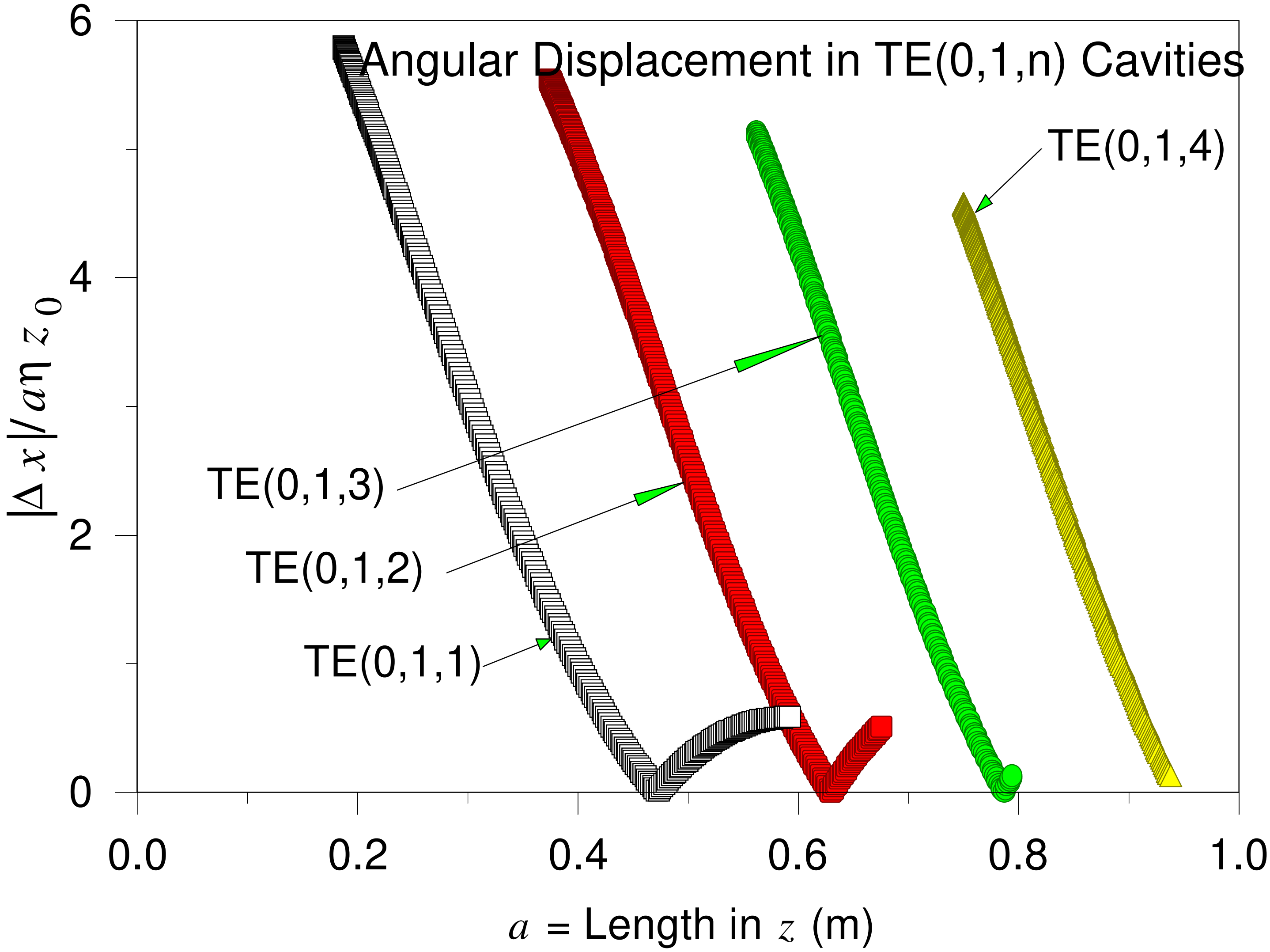
$$\text{where } f = \begin{cases} \cos, & n \text{ odd}, \\ \sin, & n \text{ even}. \end{cases}$$

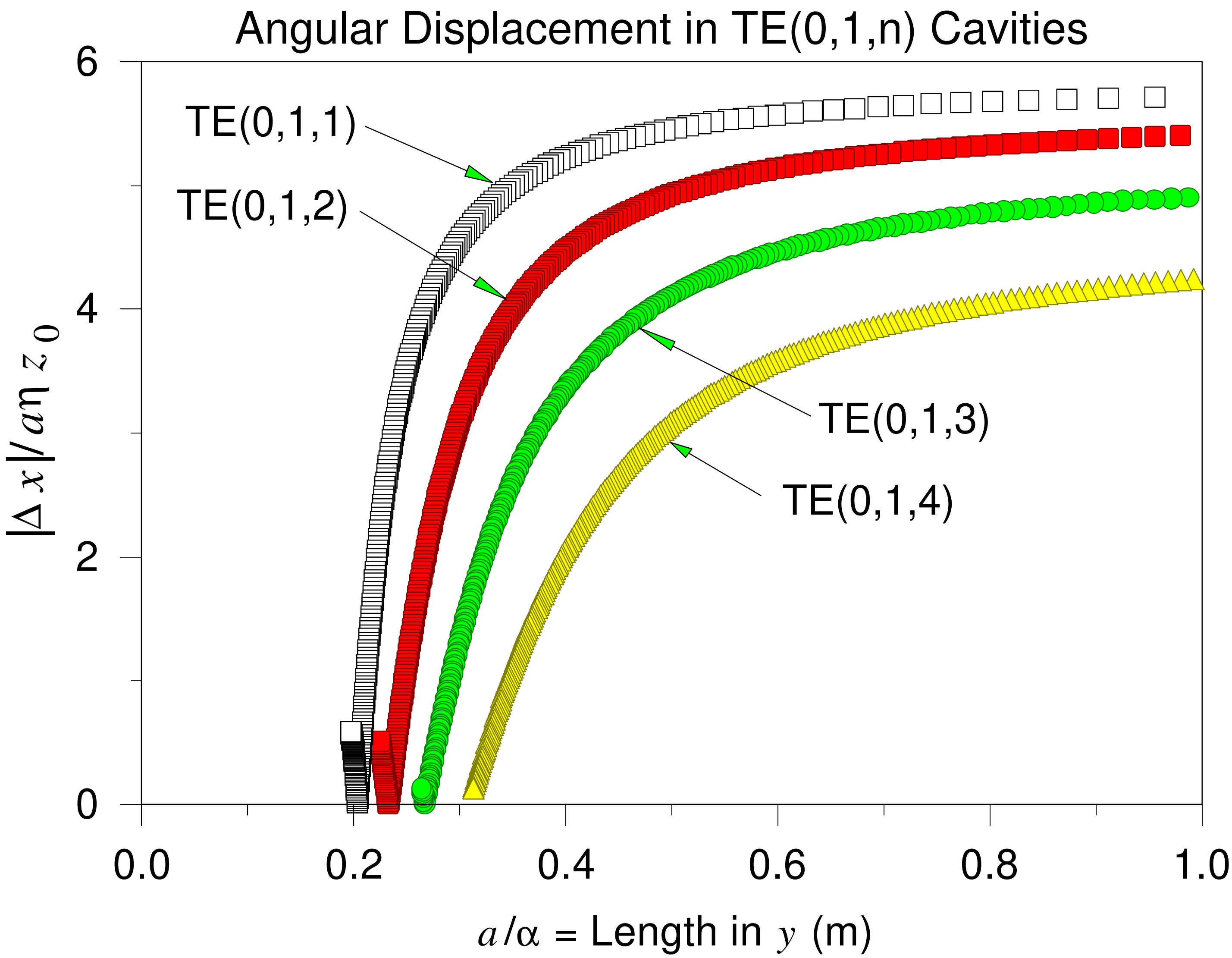

Transverse Displacement in TE(0,1,n) Cavities
TE(0,1,4)
TE(0,1,3)
TE(0,1,2)
TE(0,1,1)
$|\Delta x|/\eta z_0$
α
0
1
2
3
4
0
1
2
3

Multiple scattering will limit accuracy of Δx Measurement.

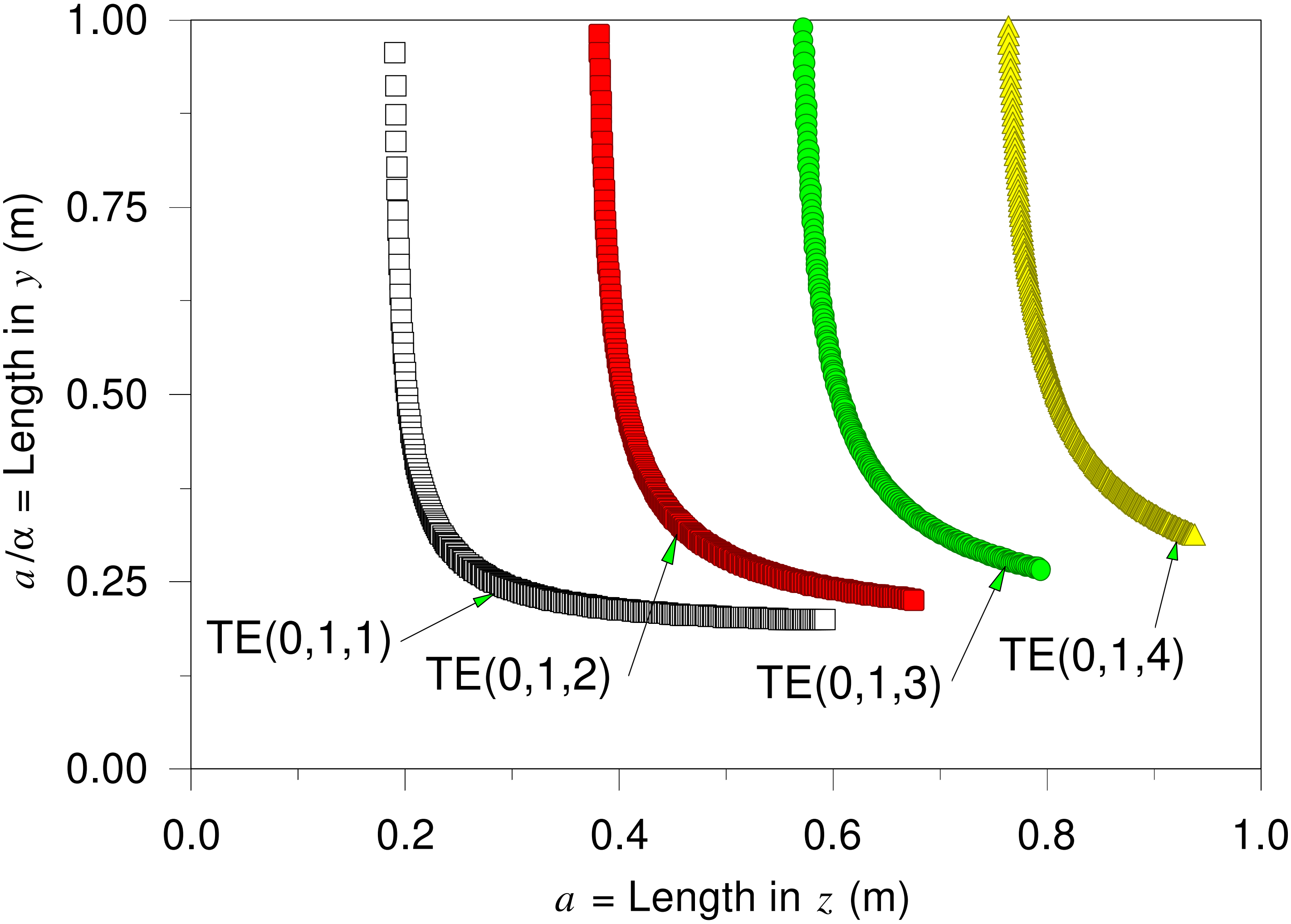
$\Rightarrow$ angular displacement $\Delta x/a$ is figure of merit.




Angular Displacement in TE(0,1,n) Cavities
TE(0,1,4)
TE(0,1,3)
TE(0,1,2)
TE(0,1,1)
$|\Delta x|/a\eta z_0$
0
2
4
6
0.0
0.2
0.4
0.6
0.8
1.0
a = Length in z (m)



Angular Displacement in TE(0,1,n) Cavities
TE(0,1,1)
TE(0,1,2)
TE(0,1,3)
TE(0,1,4)
$|\Delta x|/a\eta z_0$
a/α = Length in y (m)
0
2
4
6
0.0
0.2
0.4
0.6
0.8
1.0


1.00
0.75
0.50
0.25
0.00
0.0
0.2
0.4
0.6
0.8
1.0
a /α = Length in y (m)
a = Length in z (m)
TE(0,1,1)
TE(0,1,2)
TE(0,1,3)
TE(0,1,4)

Example of $\mathbf{TE}_{0,1,1}$ Cavity

$a = 20$ cm, $\alpha = 0.37$, $a/\alpha = 54$ cm, $\nu = 800$ MHz,

$E_0 = 40$ MV/m, $\eta = 0.0223$,

$P = 165$ MeV/c, $\gamma = 1.85$, $\beta_z = 0.85$.

$$|\Delta x| = 6.2[\mu\text{m}]\left[\frac{\Delta t}{1\ \text{ps}}\right],$$

$$\frac{|\Delta x|}{a} = 30[\mu\text{rad}]\left[\frac{\Delta t}{1\ \text{ps}}\right].$$

If $\sigma_{D,t} = \sigma_t = 40$ ps, then need

$$X_0 < \left(\frac{0.0012 \cdot 165 \cdot 0.84}{15}\right)^2 = 0.00012 \text{ radiation lengths.}$$

Thus the entrance and exit walls of a copper RF cavity should be less than 1.7 μm thick, or less than 43 μm thick if the walls are made of beryllium.

[It does not help to use mutiple cavities when multiple-scattering limited.]