STRONG FIELD QED

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A recent experiment performed in collaboration with Adrian Melissinos produced evidence for "sparking the vacuum". This work is placed in the context of an appreciation of Adrian's genealogy as well as other of his contributions to physics.

1 The Melissinoi

Some of the lore of the Melissinos clan be be traced on the internet.[1]

There are relatively few Melissinoi in the USA, with a core group in the Greek neighborhoods of New York City. Many work in the various professions, and several have web sites that indicate active involvement in community affairs.

Adrian is perhaps the only Melissinos scientist. A charming reference to his textbook on experimental physics[2] is a science-fair project by a 12-year-old girl in Alabama.[3]

The most famous Melissinos worldwide may be the poet Stavros,[4] who dispenses sandals and wisdom from his shop in the Monistraki district of Athens. Several books of his are in the Princeton University library, all in Greek. Other Greek Melissinoi include the jet skier Costas,[5] and the artist Pantelis[6] whose recent show is titled "Sensations, Obsessions and Illusions". Elsewhere in Europe are Alexandre,[7] a French architect and son of Adrian's first cousin, and Sakis[8] in Gävle, Sweden.

The Melissinos clan can be traced to the late Byzantine era in Crete, when they were prominent among the nobility of the western end of the island.[9] After Crete was ceded to Venice in 1212, the Melissinoi led several uprisings in protests of oppression by the new Venetian landlords. Adrian comments: "There is a connection of my family directly back to the Byzantines, but rather along a Peloponesus branch. The Cretan branch had to leave when the Turks came, and the settled and prospered in Cephalonia (one of the Ionian islands)."

The totem of the Melissinos clan is the bee, as indicated by the root "meliss". The Cretan Melissinoi were granted a coat of arms by the Venetians during the 13th century,[10] whose heraldic description is "gules, 3 bells sable, 6 bees or, 1, 2, 2, 1". This phrase is a line of code in the heraldic programming language, which is still functional after 800 years. The Melissinos blazon shown in Fig. 3 was produced with the software BLAZONS! 95 using the

Figure 1. Stavros Melissinos, poet and sandalmaker.[4]

Figure 2. The announcement for Pantelis Melissinos' show "Sensations, Obsessions and Illusions".[6]

above phrase as input.[11]

The earliest known member of the Greek bee people is the philosopher Melissos, who is mentioned by Aristotle as a student of Parmenides.[12] He also appears to have been the admiral of the navy of the island of Samos which defeated the Athenian fleet under Pericles in 441 BC. Melissos is no doubt Adrian's most distinguished ancestor, and closest to him in spirit, being like Adrian both a physicist and a naval officer.

Independent of his genetic link with Adrian, Melissos is important for our

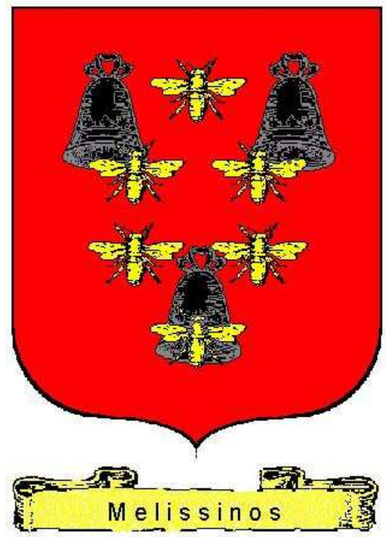

Figure 3. The Melissinos coat of arms, “gules, 3 bells sable, 6 bees or, 1, 2, 2, 1”, reconstructed via BLAZONS! 95 software.[11]

story in that he is among the first physicists to consider the nature of the vacuum, and the problem of creation of matter in terms that have contemporary resonance. Only fragments of Melissos' writings remain, but they include the powerful phrasing: “Nor is anything empty. For what is empty is nothing. What is nothing cannot be”, and “Accordingly, being was not generated, nor will it be destroyed; so it always was and always will be”.

2 Strong Field QED

2.1 Scattering Off the Vacuum

Prior to our work together on “sparking the vacuum”, Adrian performed an experiment much in the spirit of Melissos: a study of the scattering of an electron beam off the vacuum.[13] The issue is that the “vacuum” pipe of a particle accelerator is not a true vacuum, but contains room-temperature blackbody radiation as well as residual gas molecules. While effects of residual gas in vacuum pipes are well known, the blackbody radiation is usually ignored in high energy physics. Yet, in the process called “inverse” Compton scattering by astrophysicists, a beam electron can amplify the energy of a blackbody photon by a factor $4\gamma^2$, where $\gamma = E/m$ is the Lorentz factor of the beam. Thus, a room-temperature photon a energy 1/40 eV can attain energies of order 1 GeV when scattered by a 50 GeV electron beam. Such GeV photons are readily detected by the Čerenkov light emitted in their interaction with a lead glass block.

The experiment consisted of the 50-GeV LEP beam at CERN, a single lead glass block, and "no" target. The spectrum of scattering photons shown in Fig. 4 exhibits an excess at low energies, consistent with scattering off thermal photons in the "vacuum".

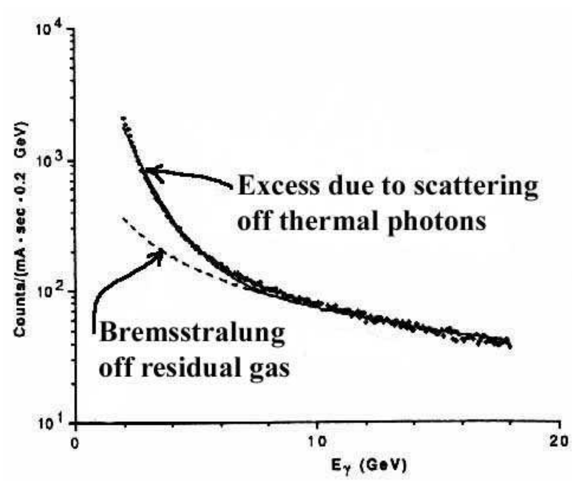

Figure 4. Scattering of 50-GeV electrons off the "vacuum" includes a signal of bremsstrahlung due to residual gas molecules, as well as "inverse" Compton scattering off thermal photons.[13]

2.2 A Path to Strong Field QED

Prior to the Maxwellian synthesis of electromagnetism, the Navier-Stokes equation of hydrodynamics was a candidate for the "theory of everything"; but it didn't predict sonoluminescence.[14] The latter is the phenomenon in which an imploding bubble of gas inside a liquid converts a large fraction of the initial acoustic energy into visible light by a process that is not well understood.[15] [It is likely that the eV-scale photons of sonoluminescence are what makes liquid nitroglycerine explode when dropped.[16]]

Several recent speculations relate sonoluminescence to QED process:

- Schwinger:[17] a bubble is an electromagnetic cavity; an imploding bubble will radiate away the changing, trapped zero-point energy. This is a dynamic manifestation of the Casimir effect.[18]
- Liberati *et al.*:[19] an imploding bubble ⇒ rapidly changing index of refraction ⇒ associated radiation. This relates to an earlier idea:
- Yablonovitch:[20] an accelerating boundary across which the index of refraction changes is a realization of the Hawking-Unruh effect, leading to conversion of QED vacuum fluctuations into real photons.

In 1974, Hawking noted that an observer outside a black hole experiences a bath of thermal radiation of temperature

$$T = \frac{\hbar g}{2\pi ck} \ , \tag{1}$$

where g is the local acceleration due to gravity.[21] In some manner, the interaction of the gravitational field with QED vacuum fluctuations produces measurable effects on an observer. Shortly thereafter, Unruh remarked that according to the equivalence principle an accelerated observer in a gravity-free region should also experience a thermal bath with temperature

$$T = \frac{\hbar a}{2\pi ck} \ , \tag{2}$$

where a is the acceleration of the observer as measured in his instantaneous rest frame.[22]

The Hawking-Unruh effect is perhaps novel, but it should be contained within the standard QED theory. Indeed, Bell noted that the incomplete polarization of electrons in a storage ring is explained in detail by Hawking-Unruh excitations.[23] Recently, Leinaas[24] and Unruh[25] have extended that argument to arbitrary g-factors for the electron, fully reproducing the intricate dependence of the polarization on g (Fig. 5).

2.3 Nonlinear QED of Strong Fields

The advice of Hawking and Unruh is that novel QED phenomena may be found in highly accelerated systems. For example, if the energy kT in eq. (2) is to correspond to the electron rest energy, then the acceleration a must be $2\pi mc^3/\hbar$. If this acceleration is caused by an electric field E, then $a = eE/m$ and we find that $E = 2\pi E_{\rm crit}$, in terms of the QED critical field strength,[26,27,28]

$$E_{\rm crit} = \frac{m^2c^3}{e\hbar} = \frac{mc^2}{e\lambda_C} = 1.3 \times 10^{16} \text{ V/cm} = 4.3 \times 10^{13} \text{ Gauss.} \tag{3}$$

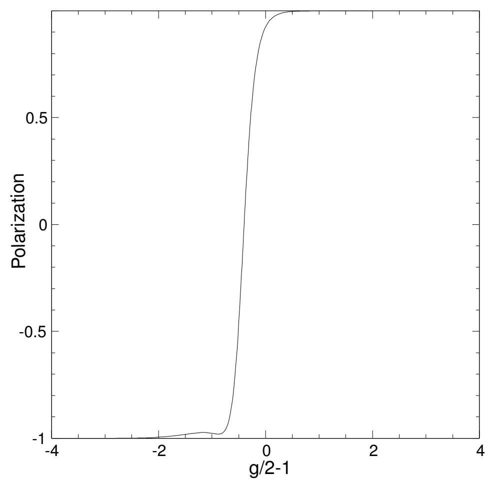

Figure 5. Calculated dependence of the residual polarization in an electron storage ring on the g-factor of the electron.[25]

Thus we are led to consider physics opportunities in very strong macroscopic electromagnetic fields. The strongest laboratory fields are found in lasers. Tabletop teraWatt lasers[29] can be focused to $> 10^{19}$ W/cm^2, $\Rightarrow E > 100$ GeV/cm, for which the photon number density exceeds 10^{27}/cm^3.

In such strong fields, the (nonperturbative) physics is described by two dimensionless measures of field strength, η and Υ:

$$\eta = \frac{e\sqrt{\langle A_\mu A^\mu \rangle}}{mc^2} = \frac{eE_{\rm rms}}{m\omega_0 c} = \frac{eE_{\rm rms}\lambda_0}{mc^2}, \tag{4}$$

governs the importance of multiple photons in the initial state, and characterizes the "mass shift":[30] $\overline{m} = m\sqrt{1+\eta^2}$.

The second parameter,

$$\Upsilon = \frac{\sqrt{\langle (F^{\mu\nu} p_\nu)^2 \rangle}}{mc^2 E_{\rm crit}} = \frac{2p_0}{mc^2}\frac{E_{\rm rms}}{E_{\rm crit}} = \frac{2p_0}{mc^2}\frac{\lambda_C}{\lambda_0}\eta, \tag{5}$$

governs the importance of "spontaneous" pair creation, where $E_{\rm crit}$ is the QED critical field strength (3).

Where can critical fields be found?

- The magnetic field at the surface of a neutron star can exceed the critical field $B_{\rm crit} = 4.4 \times 10^{13}$ Gauss.[31]

- During heavy-ion collisions where $Z_{\rm total} = 2Z > 1/\alpha$, the critical field can be exceeded and e^+e^- production is expected:[32,33] $E_{\rm max} \approx 2Ze/\lambda_C^2 = 2Z\alpha E_{\rm crit}$.
- The earth's magnetic field appears to be critical strength as seen by a cosmic-ray electron with 10^{19} eV.[34]
- The effective field between planes of a crystal can appear critical to a highly relativistic electron.[32]
- The electric field of a bunch at a future linear collider approaches the critical field in the frame of the oncoming bunch.[35]
- The electric field of a focused teraWatt laser appears critical to a counterpropagating 50-GeV electron.[36]

  The latter case was explored in a recent set of experiments in which the author collaborated with Adrian and others, the concept of which is sketched in Fig. 6.

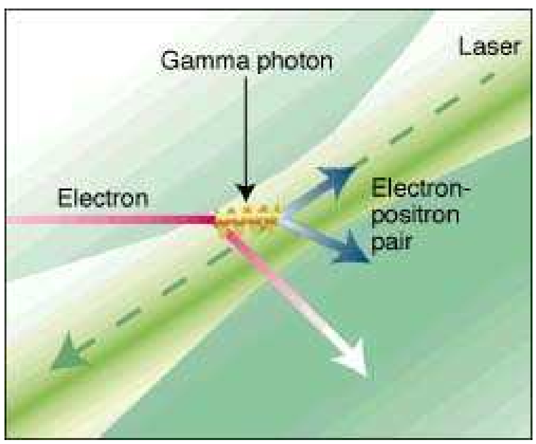

Figure 6. Scheme of probing strong-field QED in the collisions of a 50-GeV electron beam with a focused laser beam.

2.4 Physics at High η: Nonlinear Compton Scattering

The process of nonlinear Compton scattering,

$$e + n\omega_0 \to e' + \omega, \qquad (6)$$

has recently been studied for values of η up to 0.6 with apparatus as sketched in Fig. 7 and shown in Fig. 8.[37,38]

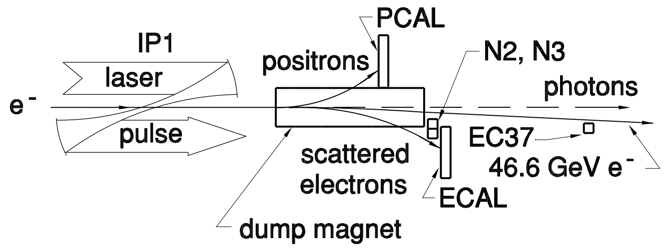

Figure 7. Layout of SLAC E-144 in which a teraWatt laser collides at 17° with a 46.6 GeV electron beam to produce backscattered photons, electrons and positrons via nonlinear QED processes.

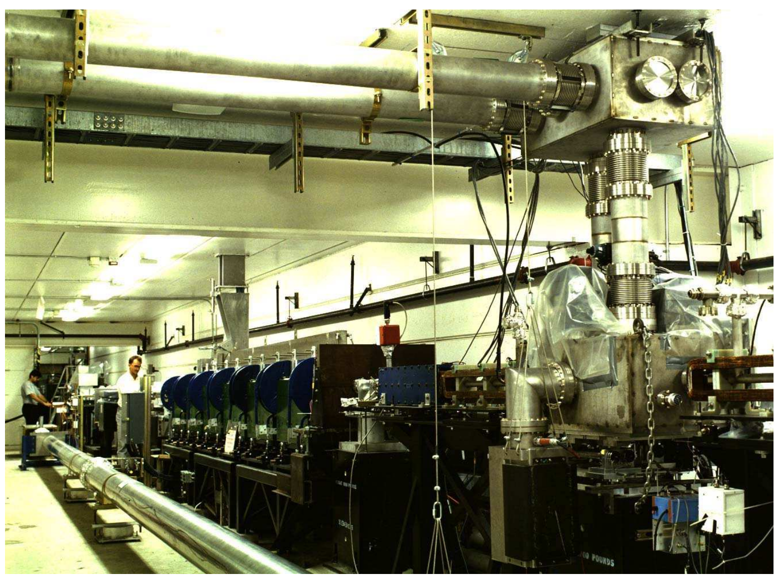

Figure 8. Photograph of the E144 apparatus in the SLAC FFTB tunnel. The electron beam enters from the right, and collides with the laser beam that is brought across the ceiling and down into the stainless steel target box. The magnets with the blue coils "dump" the unscattered electrons slightly downwards. The positron detector, PCAL, can be seen suspended from the ceiling near the center of the photograph. Wolfram Ragg is working on the electron calorimeter, ECAL, and Theo Kotseroglou is working on the electron monitor, EC37, at the left of the photograph.

The key results are summarized in Fig. 9, in which the differential rate of scattered electrons is normalized to the total rate of scattered photon electrons and plotted against laser intensity I. The minimum number n of laser photons that interacted with the electron is a simple function of the energy of the scattered electron; lower energy $\Leftrightarrow$ more photons, as labelled on the figure. The scattering rate for n-photon absorption is expected to vary as I^n, so the normalized rate varies as I^{n-1}.

The results are in good agreement with the theory based on Volkov states[39] of Dirac electrons in a plane wave,[40,41] given the fairly large systematic uncertainty in the normalization of the data, shown by the bands in Fig. 9.

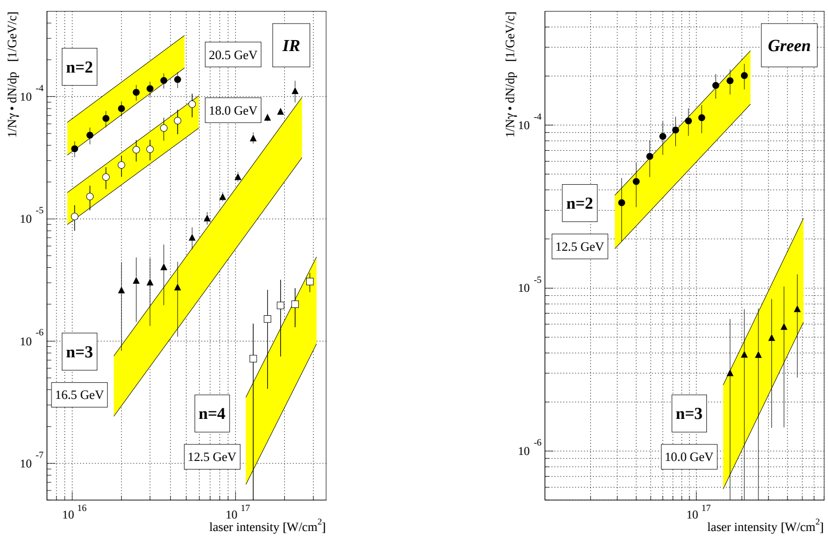

Figure 9. Normalized rate of nonlinear Compton scattering, reaction (6), as a function of laser intensity for various electron energies corresponding to different numbers of interacting laser photons. Up to four laser photons participated in a single scattering event.

Our experiment did not provide an explicit demonstration of the mass-shift effect, but this has been given in a recent experiment[42] in which a Ti:sapphire laser of invariant strength $\eta = 0.6$ was scattered off a 10 keV electron beam, with the second harmonic radiation peaking at 470 nm as shown in Fig. 10, rather than 400 nm as would be the case for $\eta \ll 1$. The mass shift $\overline{m} = m\sqrt{1+\eta^2}$ implies that the wavelength of the second harmonic scattering is $\lambda' = (1+\eta^2/2)\lambda/2$.

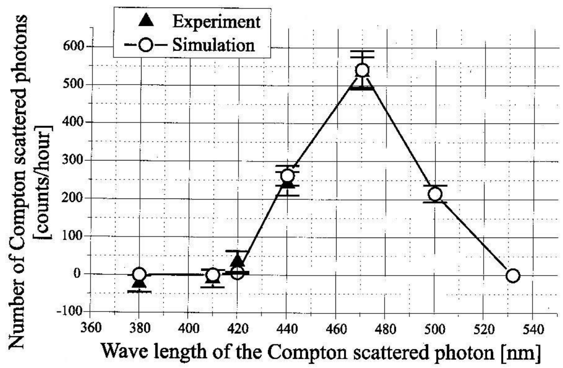

Figure 10. The wavelength of second harmonic scattering in the collision of an 800-nm Ti:sapphire laser of strength $\eta = 0.6$ with a 10-keV electron beam is shifted from 400 nm to 470 nm due to the large effective mass $\overline{m} = m\sqrt{1+\eta^2}$ of the electron inside the laser beam.[42]

2.5 Physics at High Υ*: Pair Creation by Light*

Multiphoton pair creation by light can arise from a two-step process in which a high-energy photon from reaction (6) interacts with the laser beam:

$$\omega + n\omega_0 \rightarrow e^+e^-. \tag{7}$$

This process is the strong-field variant of Breit-Wheeler pair creation.[43]

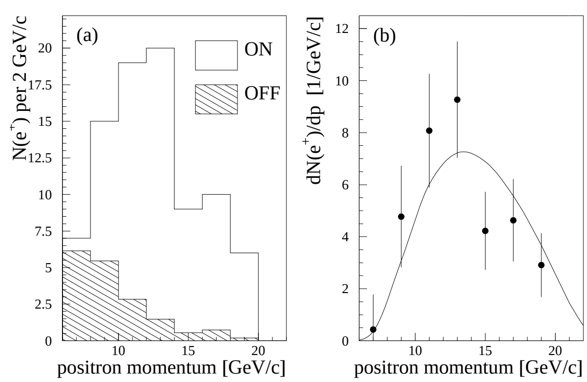

Figure 11. (a) Laser-on and -off spectra of positrons from reaction (7). (b) Subtracted spectrum. The solid line is a model calculation.

In the experiment described above, a signal of 106 ± 14 laser-induced positrons was observed after subtracting a small background of positrons from showers of lost beam electrons, as shown in Fig. 11.[44] The rate of positron production, normalized to the Compton scattering rate, is again in good agreement with strong-field QED,[36,40,41] as shown in Fig. 12. The observed rate varied as η^{2n} where $n = 5.1 \pm 0.2$ (stat.) $^{+0.5}_{-0.8}$ (syst.), $\Rightarrow$ 5 laser photons involved. In this experiment, reaction (7) was below threshold for 1-3 photons.

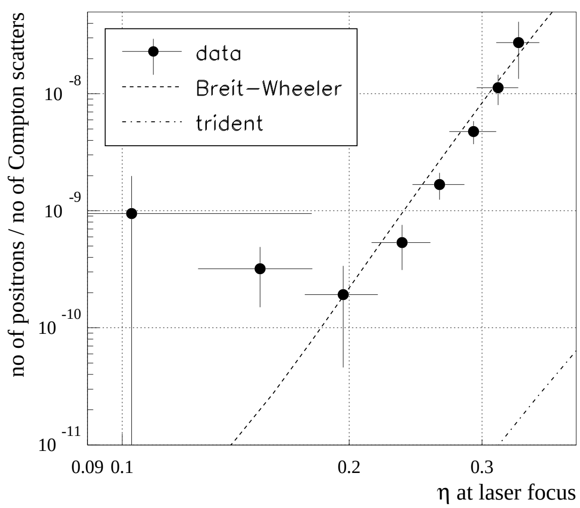

Figure 12. Rate of positron production in reaction (7), normalized to the total Compton scattering rate, as a function of laser field strength. Background from the trident process, $e + n\omega_0 \rightarrow e'e^+e^-$, is calculated to be negligible. About 2-3 $\times 10^{-10}$ positrons per Compton scatter arise from interactions of backscattered laser photons with beam gas.

The preceding discussion emphasized a particle viewpoint. It is instructive to consider a field viewpoint as well. For a virtual e^+e^- pair to materialize in a field E, the electron and positron must separate by distance d sufficient to extract energy $2mc^2$ from the field; hence, $eEd \geq 2mc^2$. The probability of a separation d arising as a quantum fluctuation is related to penetration through a barrier of thickness d:

$$P \propto \exp\left(-\frac{d}{\lambda_C}\right) = \exp\left(-\frac{2m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{2E_{\rm crit}}{E}\right) = \exp\left(-\frac{2}{\Upsilon}\right). \quad (8)$$

In a detailed analysis, $2/\Upsilon$ becomes π/Υ.[26,27,28]

When the results of Fig. 12 are plotted against $1/\Upsilon$, one finds that $R_{e^+} \propto \exp[(-1.8 \pm 0.2\,(\text{stat.}) \pm 0.2\,(\text{syst.}))/\Upsilon]$. This agreement with the barrier-penetration model justifies the interpretation of the data as "sparking the vacuum".

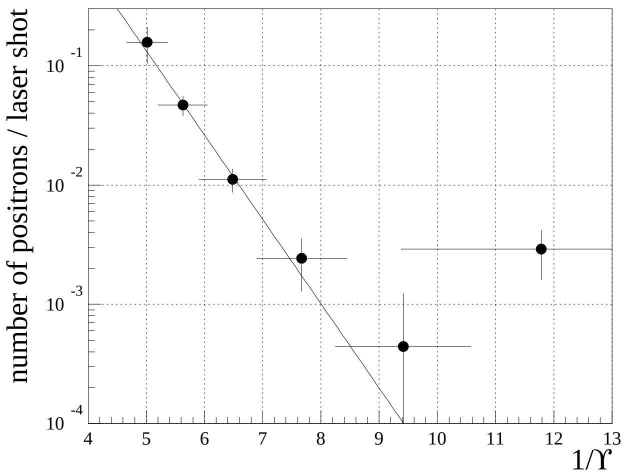

Figure 13. The observed rate of positron production *vs.* $1/\Upsilon$.

These results have led to considerable interest in the popular press, including a cartoon, shown in Fig. 14. The actual experimenters are shown in Fig. 15.

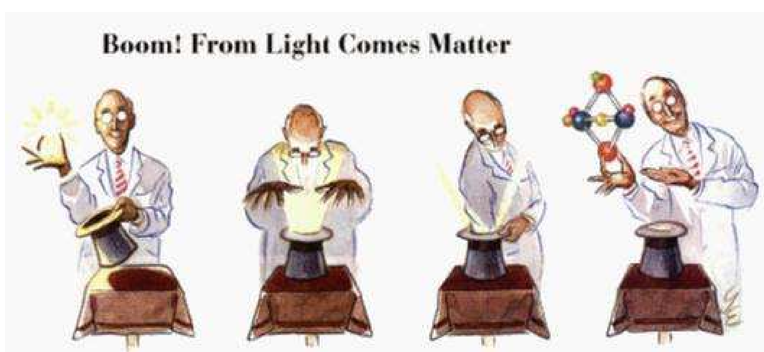

Figure 14. Cartoon of creation of matter from light, by Gil Eisner of Photonics Spectra.

Figure 15. The E-144 Collaboration. Front: Glenn Horton-Smith, Theo Kotseroglou, Wolfram Ragg, Steve Boege; back: Kostya Shmakov, Dave Meyerhofer, Charlie Bamber, Bill Bugg, Uli Haug, Achim Weidemann, Dieter Walz, Dave Burke, Jim Spencer, Christian Bula, Kirk Mc-Donald, Adrian Melissinos; not shown: Clive Field, Steve Berridge, Eric Prebys, Thomas Koffas, Dave Reis.

3 Physics with Muon Beams

Recently the author has become interested in the use of muon storage rings for muon colliders[46] and neutrino factories.[47] Here, we endeavor to catch up with the foresight of Adrian, who in 1960 was the first person to discuss the possibility of a muon storage ring in an unpublished document,[48] part of which is reproduced in Fig. 16. Adrian's pioneering work was likely the inspiration for the second study of a muon storage ring, by his Rochester colleague John Tinlot.[49]

A possible sign of progress is the ambition of a neutrino factory to store some 10^{14} muons/sec in a facility costing of order $1B. This is 100,000 times more cost effective than Adrian's original thought to store 10^5 muons/sec in a ring costing $100k.

References

1. Additional Melissiniana is to be found at my web Adrianfest web page: http://puhep1.princeton.edu/˜mcdonald/adrianfest/

The University of Rochester
Rochester 20, N.Y.

1960

Department of Physics

It is proposed to build a strong focusing ring to contain μ-mesons in a given momentum band for several π-meson lifetimes and then eject them.

Orbit radius 1.9 m
Mean radius 2.2 m

Aperture width 7 cm
Aperture height 4 cm

Maximum field 13 Kg.

Field index $n_1 = n_2 = n \simeq 15$

Betatron osc freq. $Q_r = 2.25$
$Q_z = 1.75$

Rotation frequency $\simeq$ 22 Mc /sec

Weight Fe 10 tons   Cu 2 tons

Sectors: 4 each double
Focusing order: 1/2F D 1/2F

At the Cosmotron, a fairly well focused π-beam of 10^7/pulse could be obtained for a $\Delta p/p \simeq 20\%$. Assuming that for such a beam the capture efficiency is 50%, the μ-meson beam becomes

2.2×10^4/pulse with $\Delta p/p = 10\%$

At the AGS one could expect a fairly well focused beam of π's 2×10^8 so that under the same considerations the μ-mesons are 4.4×10^5/pulse with $\Delta p/p = 10\%$

The cost is estimated to less than $100,000.

If all this does not seem too unreasonable I will proceed to calculate orbits.

Figure 16. From a note by Adrian Melissinos in 1960, proposing a muon storage ring.[48]

2. Adrian C. Melissinos, *Experiments in Modern Physics* (Academic Press, 1966).
3. Rebekah Rogers, *Using Čerenkov Radiation to Measure Potassium in Fertilizer* (March 19, 1996), http://ourworld.compuserve.com/homepages/Bill_Rogers_AL/scifair.htm
4. A Stavros Melissinos web site: http://www.athensguide.com/poet.html
5. Costas Melissinos is mentioned at http://www.tourhotel.gr/tourolympic/neo.htm
6. Pantelis Melissnos web page: http://cheapart.lrf.gr/ennotice.htm
7. An Alexandre Melissinos web page: http://www.cpod.com/monoweb/thomas/rva97/html/melissin.htm
8. Sakis Melissinos home page: http://surf.to/Sakis
9. An overview of the history of medieval Crete can be found at http://www.sfakia-crete.com/sfakia-crete/history.html
   The greatest detail in English about the Melissinoi of this era appears to be in guidebooks such as: Stephanos Psimenos, *Unexplored Crete* (Road Editions, Ltd, Athens, 1996), pp. 38-39.
10. The Melissinos coat of arms is mentioned at the Byzantine heraldry web site: http://www.heraldica.org/topics/national/byzantin.htm
11. BLAZONS! 95, heraldry software for the PC: http://www.blazons.com
12. The works of the philosopher Melissos are discussed at the sites:
    http://history.hanover.edu/texts/presoc/melissos.htm
    http://www.utm.edu/research/iep/text/presoc/presoc.htm
13. B. Dehning *et al.*, *Scattering of high energy electrons off thermal photons* Phys. Lett. **B249**, 145 (1990).
14. T. Erber, quoted by Putterman.[15]
15. S.J. Putterman, *Sonoluminescence: Sound into Light*, Sci. Am. (Feb. 1995), p. 46; R.A. Hiller, S.J. Putterman and K.R. Weninger, *Time-Resolved Spectra of Sonoluminescence*, Phys. Rev. Lett. **80**, 1090 (1998).
16. C.H. Johansson and P.A. Persson, *Detonics of High Explosives*, (Academic Press, London, 1970).
17. J. Schwinger, *Casimir energy for dielectrics*, Proc. Nat. Acad. Sci. **89**, 4091, 11118 (1992); *Casimir light: A glimpse*, **90**, 958, 2105, 4505, 7285 (1993); **91**, 6473 (1994).
18. S.K. Lamoreaux, *Demonstration of the Casimir Force in the 0.6 to 6 μm Range*, Phys. Rev. Lett. **78**, 5 (1997).
19. S. Liberati *et al.*, *Sonoluminescence: Bogolubov coefficients for the QED vacuum of a collapsing bubble* (quant-ph/9805023, 8 May 1998); *Sonoluminescence as a QED vacuum effect* (quant-ph/9805031, 11 May 1998).
20. E. Yablonovitch, *Accelerating Reference Frame for Electromagnetic*

Waves in a Rapidly Growing Plasma: Uhruh-Davies-Fulling-DeWitt Radiation and the Nonadiabatic Casimir Effect, Phys. Rev. Lett. **62**, 1742 (1989).
21. S.W. Hawking, *Black Hole Explosions*, Nature **248**, 30 (1974); *Particle Creation by Black Holes*, Comm. Math.-Phys. **43**, 199 (1975).
22. W.G. Unruh, *Notes on Black Hole Evaporation*, Phys. Rev. D **14**, 870 (1976); *Particle Detectors and Black Hole Evaporation*, Ann. N.Y. Acad. Sci. **302**, 186 (1977).
23. J.S. Bell and J.M. Leinaas, *Electrons as Accelerated Thermometers*, Nucl. Phys. **B212**, 131 (1983); *The Unruh Effect and Quantum Fluctuations of Electrons in Storage Rings*, **B284**, 488 (1987); J.S. Bell, R.J. Hughes and J.M. Leinaas, *The Unruh Effect in Extended Thermometers*, Z. Phys. **C28**, 75 (1985).
24. J.M. Leinaas, *Accelerated Electrons and the Unruh Effect* (hep-th/9804179, 28 Apr 1998).
25. W.G. Unruh, *Acceleration Radiation for Orbiting Electrons* (hep-th/9804158, 23 Apr 98).
26. F. Sauter, *Über das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs*, Z. Phys. **69**, 742 (1931); *Zum "Kleinschen Paradoxon"*, **73**, 547 (1931).
27. W. Heisenberg and H. Euler, *Folgerungen aus der Diracschen Theorie des Positrons*, Z. Phys. **98**, 718 (1936).
28. J. Schwinger, *On Gauge Invariance and Vacuum Polarization*, Phys. Rev. **82**, 664 (1951).
29. D. Strickland and G. Mourou, *Compression of Amplified-Chirped Optical Pulses*, Opt. Comm. **55**, 447 (1985).
30. T.W.B. Kibble, *Frequency Shift in High-Intensity Compton Scattering*, Phys. Rev. **138**, B740, (1965); *Refraction of Electron Beams by Intense Electromagnetic Waves*, Phys. Rev. Lett. **16**, 1054 (1966); *Mutual Refraction of Electrons and Photons*, Phys. Rev. **150**, 1060 (1966); *Some Applications of Coherent States*, *Cargèse Lectures in Physics*, Vol. 2, ed. by M. Lévy (Gordon and Breach, New York, 1968), p. 299.
31. C. Kouveliotou *et al.*, *An x-ray pulsar with a superstrong magnetic field in the soft x-ray repeater SGR1806-20*, Nature **393**, 235-237 (1998).
32. A. Schaefer, *QED in Strong Fields*, Nucl. Instr. and Meth. **B119**, 48 (1996).
33. G. Taubes, *The One That Got Away?*, Science **275**, 148 (1997).
34. I. Pomeranchuk, *On the Maximum Energy Which the Primary Electrons of Cosmic Rays Can Have on the Earth's Surface Due to Radiation in the Earth's Magnetic Field*, J. Phys. (USSR) **2**, 65 (1940).

35. P. Chen and V.L. Telnov, *Coherent Pair Creation in Linear Colliders*, Phys. Rev. Lett. **63**, 1796 (1989).
36. H.R. Reiss, *Absorption of Light by Light*, J. Math. Phys. **3**, 59 (1962); *Production of Electron Pairs from a Zero-Mass State*, Phys. Rev. Lett. **26**, 1072 (1971).
37. C. Bula *et al.*, *Observation of Nonlinear Effects in Compton Scattering*, Phys. Rev. Lett. **76**, 3116 (1996).
38. C. Bamber *et al.*, *Studies of nonlinear QED in collisions of 46.6 GeV electrons with intense laser pulses*, Phys. Rev. D. **60**, 092004 (1999).
39. D.M. Volkov, *Über eine Klasse von Lösungen der Diracschen Gleichung*, Z. Phys. **94**, 250 (1935).
40. A.I. Nikishov and V.I. Ritus, *Quantum Processes in the Field of a Plane Electromagnetic Wave and in a Constant Field. I; II*, Sov. Phys. JETP **19**, 529, 1191 (1964);
41. N.B. Narozhny *et al.*, *Quantum Processes in the Field of a Circularly Polarized Electromagnetic Wave*, Sov. Phys. JETP **20**, 622 (1965).
42. K. Matsukado *et al.*, *Toward the experimental observation of onlinear effects in laser-electron beam scattering*, Rev. Sci. Instr. **71**, 369 (2000).
43. G. Breit and J.A. Wheeler, *Collision of Two Light Quanta*, Phys. Rev. **46**, 1087 (1934).
44. D.L. Burke *et al.*, *Positron Production in Multiphoton Light-by-Light Scattering*, Phys. Rev. Lett. **79**, 1626 (1997).
45. For reports in the popular press, see
http://www.slac.stanford.edu/exp/e144/popular.html/
Presto! Light Creates Matter, Science Now Online, Aug. 20, 1997;
Conjuring Matter from Light, Science magazine, 29 Aug. 1997, p. 1202;
Scientists Use Light to Create Matter, N.Y. Times, Sept. 16, 1997;
Real Photons Create Matter, AIP Physics News, Sept. 18, 1997;
Dalla luce e nata la materia, Corriere della Sera, Sept. 21, 1997;
Materie aus Licht, Neue Zuricher Zeitung, Sept. 26, 1997;
Materie aus Licht erschaffen, Oct. 1, 1997;
Light Work, New Scientist magazine, Sept. 27, 1997;
Amerikanische Physiker schufen erstmals Materie aus reinem Licht, Die Zeit, Oct. 17, 1997;
Matter Created from Pure Light, OE Reports, No. 167, Nov. 1997;
Boom! from Light comes Matter, Photonics Spectra online, Nov. 1997, also, p. 31 of Photonics Spectra, Nov. 1997;
Matter from Light, CERN Courier, Nov. 1997, p. 4;
$E = mc^2$, *Really*, Scientific American, Dec. 1997;
Let There Be Matter, Discover magazine, Dec. 1997, p. 40;

Researchers Create Matter from Light, Laser Focus World, Dec. 1997, p. 29;
Out of Pure Light, Physicists Create Particles of Matter, SLAC Interaction Point magazine, Dec. 1997;
Let There Be Matter, a 2:04 min. interview by Karen Fox for the AIP Science Report;
Gamma Rays Create Matter Just by Plowing into Laser Light, Physics Today, Feb. 1998.

46. C.M. Ankenbrandt *et al.*, *Status of muon collider research and development and future plans*, Phys. Rev. ST Accel. Beams **2**, 081001 (1999).
47. K.T. McDonald, ed., *Expression of Interest for R&D towards A Neutrino Factory Based on a Storage Ring and a Muon Collider*, (Nov. 7, 1999); http://puhep1.princeton.edu/~mcdonald/NSFLetter/nsfmain.ps
48. The full text of Adrian Melissinos' 1960 note on muon storage rings can be viewed at http://puhep1.princeton.edu/mumu/phys/meliss1/1.html
49. J. Tinlot, *A Storge Ring for 10-BeV Muons*, SLAC-5 (Aug. 1962), p. 165.