Some Remarks on the e-Document thoughts.pdf by Bill Miller

Kirk T. McDonald
kirkmcd@princeton.edu
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The article thoughts.pdf, http://www.antennex.com/library/Nov04/Nov604/thoughts.pdf by Bill Miller, KT4YE, that appeared in the Nov. 2004 issue of *antennex* purports to give new insights into magnetic fields in low-frequency circuits that include a capacitor.

However, essentially every attempt by Miller to go beyond a "textbook" discussion of this situation is flawed by a poor choice of the kind of questions that he asks (points 1-2 below), and by serious inconsistencies of logic (points 3-8 below).

1. Miller seems concerned with "why" things happen, rather the "how".

   Such concerns are very natural, but the history of science has shown that questions of "why" are more elusive than "how".

   The domain of science is much more the "how" than the "why".

   If $A = B$, is B the reason "why" A is A?

   We can also write this as $B = A$? Then, should we conclude that actually A is the reason "why" B is B?

   And if $A = B + C$, so that $A - B = C$, then is C the reason why $A - B$ is what it is?

   Most people would realize that my above use of equations has no real meaning.

   But when Miller contemplates the Ampère-Maxwell equation

   $$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\,, \tag{19}$$

   he seems to ask "WHY a varying $\mathbf{E}$ field should cause a magnetic field to occur?"[1]

[1]p. 5 of thoughts.pdf. Miller omits to explain the relation between the electric field $\mathbf{E}$ and the electric displacement $\mathbf{D}$ that appears in his eq. (19). A more logical line of thought would note that Miller's equation (17), $\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_v$, is the only other relation given for the vector field $\mathbf{D}$, and hence we might infer that $\mathbf{D}$ is related to electric charge, so that $\partial \mathbf{D}/\partial t$ is related to the time rate of change of electric charge, which is a lot like electric current. So, another way of looking at eqs. (17) and (19) is that the magnetic field $\mathbf{H}$ is related to electric current, which is what Ampère was saying all along.

Maxwell supplemented Miller's eqs. (16)-(19) with the insight that in simple media, such as vacuum and copper wires, $\mathbf{D} = \epsilon_0 \mathbf{E}$ and $\mathbf{B} = \mu_0 \mathbf{H}$. In this case, since $\epsilon_0 \mu_0 = 1/c^2$ where c is the speed of light,

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho_v}{\epsilon_0}\,, \tag{17$'$}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}\,. \tag{19$'$}$$

Then, taking the curl of eq. (19′), and using Miller's eqs. (16) and (18), we find that

$$\boldsymbol{\nabla}^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = \mu_0 \boldsymbol{\nabla} \times \mathbf{J}.$$

One could equally well write Miller's eq. (19) as

$$\frac{\partial \mathbf{D}}{\partial t} = -\boldsymbol{\nabla} \times \mathbf{H} + \mathbf{J}, \tag{19$''$}$$

and claim that the question is: "Why do magnetic fields (and electric currents) cause time-varying electric displacement (*i.e.*, displacement currents)?

My point is that Maxwell's equations (and all other equations) do NOT explain "why" things happen, but they can help us understand "how" things happen.

2. Maxwell's equations (Miller's eqs. (16)-(19)), are differential equations for fields.

   They were designed to avoid the question of action at a distance, *i.e.*, how the situation here is affected by the situation over there. They are meant to provide a local view of the universe, *i.e.*, whatever happens here is related to other effects right here, not over there.

   In the local view, we say that Miller's eq. (19) gives us insight as to how a time-varying electric displacement here is related to the magnetic field right here.

   But Maxwell is very flexible. He doesn't insist that you only adopt the local view. You are free to integrate his equations to find forms that better match the view of action at a distance. The result at the end of footnote 1 is an example of this way of looking at things.

   Miller remarked that Maxwell's differential equations can be put into integral form, which I present as

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{Area}, \tag{16*}$$

$$\int \mathbf{D} \cdot d\mathbf{Area} = \int \rho_v d\mathrm{Vol} \tag{17*}$$

$$\int \mathbf{B} \cdot d\mathbf{Area} = 0, \tag{18*}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int \mathbf{J} \cdot d\mathbf{Area} + \frac{d}{dt} \int \mathbf{D} \cdot d\mathbf{Area}. \tag{19*}$$

   These relations are not local, but relate fields at some places to fields in other places.

   But great caution is required in interpreting these equations as explaining "why" fields in some places are caused by fields in other places.

   Because of the freedom to choose the surfaces and volumes of integration, these equations do NOT make unique associations between fields at one place and those at another.

Many people find this provides a useful way of thinking about the relation between currents and magnetic fields. The solution to this equation is

$$\mathbf{B}(\mathbf{x}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}', t' = t - s/c) \times \hat{\mathbf{s}}}{s^2} d^3\mathbf{x}' + \frac{\mu_0}{4\pi c} \int \frac{\dot{\mathbf{J}}(\mathbf{x}', t' = t - s/c) \times \hat{\mathbf{s}}}{s} d^3\mathbf{x}',$$

which relates the magnetic field $\mathbf{B}$ ONLY to electrical current density $\mathbf{J}$.

This ambiguity is in the nature of field equations, and is not peculiar to Maxwell's equations for electromagnetism.

I interpret much of Miller's concern with Maxwell's equation as unease with the intrinsic ambiguities in the field point of view, which does not/cannot give unique answers to questions of "why".

3. Now for more specific comments.

On p. 9 of thoughts.pdf Miller notes that electric currents are due to motion of electric charges. But it does NOT follow that currents in the plates of a charging/discharging capacitor are anything like those shown in Miller's Fig. 5.

On p. 2 Miller says "Let us also recall some basic assumptions. We shall neglect fringing fields. All conductors are perfect."

By p. 9 Miller appears to have forgotten his own assumptions.

Perfect conductors can only support surface currents, *i.e.*, motion of charges parallel to the surface of the conductor. Whereas in Miller's Fig. 5 shows only currents that are perpendicular to the surface of the "conductor".

Thus, Miller's Fig. 5 cannot be referring to a perfect conductor.

It also is NOT referring to a GOOD, but not perfect, conductor such as copper.[2]

In good conductors the currents are not confined to the surface, but can penetrate some distance into the conductor (called the skin depth). In this case the currents are not purely parallel to the surface of the conductor, but include TINY components perpendicular to the surface. Only in extremely sophisticated analysis (which Miller's is not) do these tiny perpendicular currents play a role.[3]

On p. 9 Miller also says "It can be shown (Didn't you just love it when your instructor said that in school!) that for current in a wire, displacement current is negligible."

If he had been paying attention to his instructor, Miller would have learned that in the approximation where the displacement current in wires is negligible, then THE PER-

[2]Miller's Fig. 5 most nearly resembles so-called polarization currents ($\mathbf{J}_{\rm pol} = \partial\mathbf{P}/\partial t$, where $\mathbf{P}$ is the dielectric polarization vector) inside an insulator, rather than conduction currents inside a metal. If the gap of Miller's capacitor contained a dielectric, as most capacitors do, then his discussion of sources of the magnetic field in and around the capacitor should include the effect of these polarization currents. Since $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$, Maxwell's displacement current includes the effect of these polarization currents.

[3]Analyses that include the perpendicular components of currents in good conductors have been given for over a century. Many of these analyses come under the heading of "eddy currents", which are important in the operation of transformers. An extensive, high-level discussion of such things in given in chap. 10 of *Static and Dynamic Electricity*,$^{\rm rd}$ ed. by W.R. Smythe.
http://kirkmcd.princeton.edu/examples/EM/smythe_68.pdf
Discussion of the small effect of the (small) perpendicular components of time-dependent currents in wires of a transmission line is given in p. 535 of *Electromagnetic Theory* by J.A. Stratton.
http://kirkmcd.princeton.edu/examples/EM/stratton_electromagnetic_theory.pdf
See also p. 12 of my handwritten notes
http://kirkmcd.princeton.edu/examples/ph501lecture13/ph501lecture13.pdf
and remember that $\mathbf{J} = \sigma\mathbf{E}$ relates current density to electric field.

PENDICULAR CURRENTS, which he calls "orthogonal currents", $\mathbf{J}_0$, ARE ALSO NEGLIGIBLE.

Hence, all Miller's following analysis is based on a set of inconsistent assumptions.

4. Another badly inconsistent statement is made at the top of p. 11: "Looking in 3 dimensions. Let us assume that the capacitor plates are infinitely thin. Then we must recall that a capacitor plate has two sides. The applied voltage will act on each side in an equal manner. But the direction of surface flow is in opposite directions on either side of the plates. Consequently, the magnetic field at the perimeter of the plate at the gap will be in opposition to the magnetic field on the source side. These two fields will cancel, leaving a NET MAGNETIC FIELD of zero!"

   Miller implies that the currents on the two sides of the capacitor plate are equal and opposite. If so, then the situation would be the same as if there were no capacitor plates present, and the leads wires simply have a small gap between them.

   That is, Miller implies that all the charge on a capacitor is concentrated at the center of the insides of the plates. This contradicts his assumption that we can neglect fringe fields, so that the charge on the capacitor plates is spread uniformly over their insides, with no charge on their outsides.[4]

   Miller has gotten into trouble by using an inappropriate mix of assumptions that the capacitor is a perfect conductor (p. 11), and that the capacitor has an internal resistance and hence internal currents (p. 9).

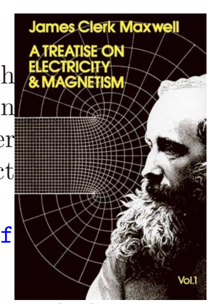
   If a capacitor were made of a perfect conductor, then indeed we would say that the current flows from the wire lead (at the center of the outside of the plate),[5] outwards to the rim of the plate an then inwards on the inside of the plate.

   There is no charge accumulation on the outside of the plate, so the current $I_o(r)$ (that crosses a circle of radius r on the outside of the plate) is independent of radius,

$$I_o(r) = I,$$

   where I is the current in the wire lead. But on the inside of the plate, the current $I_i(r)$ leads the a uniform accumulation of surface charge. As shown in eq. (29) of my note

[4] An interesting question left open in introductory courses on capacitors is how much charge is on the outsides of the plates. An answer to this ELECTROSTATIC question has been given by Maxwell, and the related figure appears on the cover of the Dover paperback edition of his *Treatise*. My handwritten notes on this fascinating subject appear on pp. 12-13 of
http://kirkmcd.princeton.edu/examples/ph501lecture13/ph501lecture6.pdf



[5] Miller's reconfiguring of the capacitor to look like a termination of a transmission line, with the capacitor leads connects to the rim of the capacitor rather than its center, breaks the symmetry of the problem so badly that elementary arguments cannot get the details right. So I revert to the simpler assumption that the wire leads are connected to the centers of the capacitor plates.

http://kirkmcd.princeton.edu/examples/displacement.pdf, this implies that the inside current has the form

$$I_i(r) = A - I\frac{r^2}{R^2},$$

where R = radius of capacitor and A is a constant to be determined. At the rim of the plate, the outside current reverses direction and becomes the inside current. Hence,

$$I_i(R) = A - I = -I_o(R) = -I,$$

which tells us that $A = 0$, so that

$$I_i(r) = -I\frac{r^2}{R^2}, \qquad \text{and} \qquad I_{\rm tot}(r) = I_o(r) + I_i(r) = I\left(1 - \frac{r^2}{R^2}\right).$$

The currents on the insides and the outside of a perfectly conducting capacitor do NOT cancel. The nonzero net current makes a significant contribution to the magnetic field![6]

5. Other inconsistent statements occur on p. 11:

   "Please note that the movement of sub-surface charges is not an **E**-Field phenomenon, but a Voltage phenomenon."

   "...in this solid structure, there is no magnetic field!"

   If there are currents within the bulk of a metal, as implied in the first quotation, then the metal is NOT a perfect conductor, and there is a non-zero magnetic field inside the metal.

   In contrast, if the metal is a perfect conductor then indeed there is no magnetic field in its interior, but neither are there any currents in the interior.

   These mis-statements echo an email exchange of over a year ago at which time I pointed out that Miller incorrectly assumes that there is no magnetic field inside current-carrying wires. Miller neither understood then nor now the difference between situations in which there are or are not magnetic fields inside conductors.

   If a wire carries a steady or low-frequency current I, then the current is uniformly distributed over the cross section of a wire. An elementary application of Ampère's law shows that the (azimuthal) magnetic field H inside the wire of radius R is given by

$$H = \frac{Ir}{2\pi R^2},$$

   for $r < R$. I am led to conclude that Miller does not understand how currents flow in wires, or that he is willfully disregarding one of the most elementary consequences of Ampère's law.

   It is true that there is no magnetic field inside a perfect conductor, in which case the current lies entirely on the surface and there are no "orthogonal" currents. By p. 11

[6]Various ways of thinking about how those surface currents are related to the magnetic field in and around the capacitor are presented in the Appendix to my note displacement.pdf (linked above).

Miller's assumptions are so unclear that the reader can't tell which case he is talking about.

This highlights a pedagogic dilemma about which assumptions to make.

(a) *Assume perfect conductors. This is sufficient for most discussions of antennas, where the radiation resistance is large compared to the ordinary resistance. But wires subject to low-frequency voltages do not behave like perfect conductors, so this is not the most useful assumption for examples like that of Miller which are related to almost DC behavior of circuits.*

(b) *Assume good conductors with skin depth small compared to the wire diameter. This allows a first approximation to the ordinary resistance encountered by high-frequency currents in conductors. But this is still not the appropriate assumptions for analyses like that of Miller.*

(c) *Assume good conductors with skin depth large compared to the wire diameter. Also, ignore the displacement current inside the wires. Then, the conduction currents are uniform across the wire, and the currents are purely longitudinal. This is the appropriate assumption for the kind of example that Miller is considering. But in this approximation, there are NONE of the effects of transverse currents that he is apparently so concerned with.*

Note that in approximation (c), the current does not flow on the surface of the capacitor plates, but inside them. In this case, the current flows radially outwards from the center of the capacitor according to the expression for $I_{\rm tot}$ given on *p. 4.*

(d) *Assume poor conductors, so that one must take into account the displacement current inside the conductors, and the currents inside the conductors have both transverse and longitudinal components. This is NOT a very useful assumption for understanding of antennas, wires and capacitors. The corrections due to the nonzero displacement current and the related transverse currents are tiny and messy. However, if we want to understand the behavior of waves inside, say, glass lenses, this is the appropriate approximation. In this case there are typically NO conduction currents, but one has to learn to deal with yet another kind of current call the polarization current, $\mathbf{J}_{\rm pol} = \partial\mathbf{P}/\partial t$, where $\mathbf{P}$ is the dielectric polarization vector (field). Or, if we are dealing with magnetic materials, we must also include the so-called magnetization current, $\mathbf{J}_{\rm mag} = \boldsymbol{\nabla}\times\mathbf{M}$, where $\mathbf{M}$ is the magnetization density.*

The full version of Maxwell's view of currents is that

$$\mathbf{J}_{\rm tot} = \mathbf{J}_{\rm conduction} + \epsilon_0 \frac{\partial\mathbf{E}}{\partial t} + \frac{\partial\mathbf{P}}{\partial t} + \boldsymbol{\nabla}\times\mathbf{M},$$

which obeys $\boldsymbol{\nabla}\cdot\mathbf{J}_{\rm tot} = 0$. Experts will note that $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$ so that the $2^{\rm nd}$ and $3^{\rm rd}$ terms of $\mathbf{J}_{\rm tot}$ can be combined into $\mathbf{J}_{\rm displacement} = \partial\mathbf{D}/\partial t$.

However, inclusion of dielectric and diamagnetic effects in discussions of wires and capacitors distracts from their simplicity.

Hence, I recommend assumption (c), rather than (d), for discussions of how electric and magnetic fields are related to conduction currents and displacement currents. in and around wires and capacitors.

The above assumptions tacitly include the assumption that we can ignore fringe fields. This is sufficient for introductory discussions.

6. On p. 12 Miller states that "According to classical thinking, we still need to explain how magnetic fields are continuous through the gap. And, of course, the explanation for this phenomenon has always been that the generation of a magnetic field happens directly from an electric field without the participation of charges."

   This harks back to my point 1, that Miller does not appear to appreciate the spirit of the field view of electromagnetism, in which one has multiple choices of explanations. One can "always" relate magnetic fields only to conduction currents, but then the currents must includes those over "there" as well as those "right here". Or, one can relate the magnetic field "here" only to other effects "right here", in which can one must consider both conduction currents and displacement currents.[7]

7. At the bottom of p. 12 Miller says "It appears that continuous magnetic fields only occur in the presence of charge flow."

   First, there are no other kinds of magnetic fields than "continuous magnetic fields."

   Second, what does "in the presence of charge flow" mean? When I receive a radio message from say, South Africa, by detecting its magnetic field in a loop antenna, am I "in the presence" of the charge flow that created the message?

   Not directly, since that charge flow occurred in South Africa. But I am sensing an indirect effect of the charge flow, as communicated to me by the "displacement" current that fills up the space between me and the broadcast antenna in South Africa. Of course, if one wants to be a bit mystical, one can say that a radio message puts one in contact with remote presences....

8. Another inconsistent statement occurs on p. 13 (but then, almost all statements in thoughts.pdf are inconsistent): "Consequently, it would appear to be a defensible postulate that the Poynting Vector, equation 24, may be restated as:

$$\mathbf{S} = \mathbf{H}_{TA} \times \mathbf{H}_{TR}. \tag{25}$$

   This implies that radiation is strictly a function of interacting magnetic fields. Electric fields play no part."

   This statement is nonsense from several points of view.

   In the stated approximation of p. 2, $\mathbf{H}_{TA} = 0$, since $\mathbf{H}_{TA}$ is the part of the magnetic field due to the negligible transverse currents in wires. Therefore, Miller's $\mathbf{S}$ vanishes.

[7]Many intermediate views are possible as well, some of which are discussed in the Appendix to my note displacement.pdf

But lurking in Miller's eq. (25) is another form of the misuse of equations referred to in my point 1 above.

The usual wave of thinking about radio waves in the far zone is that the electric and magnetic fields are related to the (unit vector) direction of propagation $\hat{\mathbf{k}}$ by

$$\mathbf{E} = 377\Omega\ \mathbf{H} \times \hat{\mathbf{k}}, \qquad \mathbf{H} = \frac{\hat{\mathbf{k}} \times \mathbf{H}}{377\Omega}.$$

Hence, the Poynting vector $\mathbf{S}$ can be written various ways:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = 377\Omega\ H^2\ \hat{\mathbf{k}} = \frac{E^2}{377\Omega}\hat{\mathbf{k}}.$$

However, we CANNOT conclude that the 2nd form of this equation tells us that the Poynting vector is due only to the magnetic field, or that the 3rd form of the equation tells us that it is due only to the electric field.

Thus, eq. (25) is just one of many examples in thoughts.pdf of the abuse of equations.

Miller's article thoughts.pdf is no longer available on the internet. Two related articles by Miller are http://kirkmcd.princeton.edu/examples/EM/miller_antennex_1102.pdf http://kirkmcd.princeton.edu/examples/EM/miller_antennex_0407.pdf