Analytic Stress Calculations for a Stepped Front Endplate

K.T. McDonald

Princeton U.

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Method

Use equations from Table 24, Case 2 of Roark and Young.

Evaluate the F_i and G_i functions separately on the two different steps.

Require continuity of deflection, slope, bending moment and shear force at the step.

Enforce conditions that $y_{\text{outer}} = y_{\text{inner}}$ and $M_{r,\text{outer}} = 0$ for a simple support there to determine slope$_{\text{inner}}$, $M_{r,\text{inner}}$ and shear$_{\text{inner}}$.

Then all values are readily determined as a function of radius.

Verify equations by using same thickness on both steps,
$\Rightarrow$ detailed agreement with Roark's examples.

Model Assumptions

The model is of a plate with no holes.

Use modulus for Al of $7/1.6 \times 10^{10}$ Pa, reducing by 1.6 for holes.

Multiply stresses by 2.5 to account for the stress concentration around holes (Roark, Table 37, Cases 6c & 7b).

Inner radius = 0.236 m.

Outer radius = 0.809 m.

Plate Simply Supported at Both Inner and Outer Radii

r(step) (cm)	Thick Thickness (mm)	Thin Thickness (mm)	Peak Deflection (mm)	Peak Radial Stress (MPa)
–	12	12	3.52	71.9
47	24	12	2.04	53.8
47	24	11	2.44	59.5
47	24	10	2.94	65.6
63	24	13	0.77	47.1
63	24	12	0.87	54.6
63	24	11	1.04	63.9
63	24	10	1.29	75.4

Plate Fixed at Inner Radius, Simply Supported at Outer Radius

r(step) (cm)	Thick Thickness (mm)	Thin Thickness (mm)	Peak Deflection (mm)	Thick Peak Radial Stress (MPa)	Thin Peak Radial Stress (MPa)
–	12	12	1.83	122.3	46.5
47	24	12	0.80	54.8	27.8
47	24	11	0.91	58.1	30.6
47	24	10	1.03	61.4	34.2
63	24	12	0.52	41.9	35.6
63	24	11	0.62	44.3	40.5
63	24	10	0.74	47.8	45.4

Remarks

1. We could reduce the peak stress by about 25% by using a step.
2. There is only a slight difference in the peak stress for a step at 47 cm compared to one at 63 cm.
3. The peak deflection is less for a step at 63 cm than one at 46 cm. This effect is more pronounced for simple supports at both inner and outer radii.
4. For a step at 47 cm there is no overall stress reduction in fixing the slope at the inner radius, as compared to a simple support there.
5. For a step at 63 cm there is a 25% stress reduction in fixing the slope at the inner radius, as compared to a simple support there.
6. For a step at 63 cm down to 13 mm a simply supported plate gives very similar performance to a step down to 10 mm for a plate fixed at the inner radius.
7. A step at 63 cm and a fixed slope at the inner radius offers

the greatest advantage in stress reduction and in minimizing deflection.

8. If we choose a step at 47 cm, an inner stiffening ring would be justified on the basis of the reduced deflection, not reduced stress.