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Options for the BaBar Drift Chamber Front Endplate

Abstract

This note summarizes estimates for thickness and deflection of flat and curved aluminum front endplates and flat carbon fiber endplates.

1 Flat Endplate

In view of the decision to use a flat aluminum endplate at the rear of the chamber, it is natural to ask about the merits of a flat aluminum plate at the front as well.

1.1 Supported at Outer Edge Only

1.1.1 Aluminum

Recall that the present design of the rear endplate is that it be 3.2 cm thick. As summarized in my note TNDC-96-20, if the 3500-kgf wire load if supported only at the outer edge of the plate, the deflection at the inner edge is 3.3 mm, and the corresponding peak radial and tangential stresses are 6.2 and 23 MPa, respectively. For comparison, the ultimate strength of 2024/T4 aluminum is 400 MPa. This endplate presents 32% of a radiation length at normal incidence, which is high.

In calculations below we downrate the ultimate strength of a BaBar aluminum endplate by the largest fraction of material lost to holes on any circle. On a zipper layer there are three 2.5-mm diameter feedthrough holes in close proximity per 18.5-mm-wide cell, so about 40% of the aluminum is lost to holes. Under extreme stress the endplate would tear along the zipper layer like a postage stamp tears along its perforations. Therefore we use $(0.6)(400) = 240$ MPa as the effective ultimate strength for the endplate.

If we take the maximum acceptable stress as 1/10 of the ultimate strength, *i.e.*, 24 MPa, then we understand that the thickness of 3.2 cm was chosen to provide a factor of 10 safety against tearing of the endplate.

In sum, a flat endplate supported at the outer edge only and having a deflection smaller than the wire elongation is thick in terms or radiation lengths.

1.1.2 Carbon Fiber

The thermoplastic carbon-fiber epoxy material under consideration by BaBar reorted has a modulus about 0.7 times that of aluminum. I am not sure of its ultimate strength, but will assume it to be also 0.7 times that of aluminum, *i.e.*, 280 MPa.

If the load and thickness of the endplate remain the same, but the modulus is changed, the stresses remain the same while the displacements scale inversly with the modulus. Thus a 3.2-cm-thick carbon fiber endplate supported at the outer radius only under a 3500-kg wire load would have a peak tangential stress of 23 MPa and a deflection of 4.7 mm at the inner radius.

If we take as a design limit a peak stress 1/10 of the ultimate strength we must limit the stress to $(0.6)(28) = 16.8$ MPa, where again the factor of 0.6 comes from downrating the strength along a zipper layer.

For a flat plate of thickness t, deflections scale as $1/t^3$ and stresses scale as $1/t^2$.

Hence the thickness of a carbon-fiber endplate would have to be increased to $3.2\sqrt{23/16.8} =$ 3.7 cm (about 15% of a radiation length) to keep the peak stress below 16.8 MPa. The deflection at the inner edge would then be $3.3/0.7/(23/16.8)^{3/2} = 2.9$ mm.

1.2 Supported at Both Inner and Outer Edges

1.2.1 Aluminum

If instead, the wire load is supported at both the inner and outer edge of the plate, then the maximum deflection of a 3.2-cm-thick Al endplate is 118 μm, and the peak radial and tangential stresses are 3.5 and 3.0 MPa, respectively.

Thus an Al plate with peak radial stress of 24 MPa should have thickness 1.2 cm (say $0.5''$), and the peak deflection would be 2.2 mm. This case is the most plausible to me of those considered thus far, and would present 12% of a radiation length at normal incidence. Should this option be chosen, it would make sense to reduce the thickness of the rear endplate to be the same as that for the front.

1.2.2 Carbon Fiber

A 1.4-cm-thick carbon fiber endplate supported at both inner and outer radii would have a peak stress of 16.8 MPa, and a maximum deflection of 2.0 mm. This plate presents 5.6% of a radiation length.

If this option were chosen and the thickness of the read aluminum endplate reduced to 1.4 cm also, the peak deflection of the latter would be 1.4 mm.

2 Curved Endplate

This option was introduced in my note TNDC-96-22. There we found that a uniform wire load per unit area could be supported by an endplate under tension only if the shape of the endplate is a certain cubic of revolution.

2.1 Supported at Both Inner and Outer Edges

The example in the note TNDC-96-22 was for this case.

2.1.1 Inward Curve

In the example given there, the slope of the curve is 1 at the outer radius and 0.667 at the inner radius. The total tension on the inner edge is 24,750 N for 3500 kgf wire load supported 40% at the inner edge. Then the plate could have a thickness of only 700 μm Al if the peak stress (at the inner edge) is 24 MPa.

The sagitta of cubic is 12 cm.

The endplate will stretch under the wire load. If the (cubic) curve that describes the endplate has length l and stretches by amount Δl then the resulting deformed curve has sagitta $s = l\sqrt{\Delta l/l}$ with respect to the undeformed curve. This assumes the endpoints of the cubic remain fixed. Due to the tension the stretch along the cubic curve is

$$\frac{\Delta l}{l} \approx \frac{T}{2\pi(r_2 - r_1)tE},$$

ignoring the resistance to stretching due to hoop stresses. With $r_2 - r_1 = 0.57$ m, taking modulus E to be 6×10^{10} including a 15% downrating for holes, and using $T = T_{\max} = 24,750$ N we have

$$s[\mathrm{mm}] = \frac{6.3}{\sqrt{t[\mathrm{mm}]}}.$$

For example, if we wish the deflection of the endplate to be only 3.15 mm we would need thickness $t = 4$ mm. *Since this ignores the favorable effect of hoop stresses a more complete calculation using FEA methods should be performed. See note TNDC-96-25 for such a calculation, which indicates that the above estimate of the deflection is much too high.*

This is the best solution if the very thinnest endplate is desired.

2.1.2 Outward Curve

While the inwardly curved plate is mechanically sound, it limits the track acceptance somewhat at intermediate radii due to the 12-cm sagitta of the curve. [Although with the demise of the aerogel the drift chamber gains about 20 cm of track length, so an inward sagitta of 12 cm for a plate close to the CsI calorimeter results in track lengths nowhere shorter than in the current BaBar baseline design.]

So we can also consider an outwardly curve plate, in which the load is supported by purely compressive forces if the same cubic is used, but with the opposite sign.

However, domed structures are subject to collapse. A semi-empirical result is that the buckling occurs at load pressures of about

$$P_{\max} = 0.5E\left(\frac{t}{r}\right)^2, \qquad \text{(Timoshenko)}$$

where r is a relevant radius of curvature (p. 320 of *Thin Shell Concrete Structures*, by D.P. Billington (McGraw-Hill, 1982)).

The typical radius of curvature for the cubic curve in the example is 40 cm. We take the modulus E of aluminum as 6×10^{10} Pa (downrating the nominal value by 15% for holes). A 3500-kgf wire load corresponds to a pressure of 18,000 Pa. If we design for buckling at 10 times the nominal load, then the thickness of the outwardly curved plate must be 1.0 mm, slightly thicker than the case of an inwardly curved plate with 10 times safety factor against tearing.

The deflection under load would be essentially the same as for the case of inward curvature, so again a plate of 4-mm thickness would deflect by about 3 mm under the wire load.

This is the best solution if one wishes to have maximum track length.

2.2 Supported at Outer Edge Only

If the endplate is to be under pure tension and unsupported at one edge, the slope of the curve must be zero at this edge. But the deflection of the plate under the wire load will cause a nonzero slope there. Hence the pure-tension concept is not consistent with plates supported at one edge only.

Of course, such plates can still be contemplated. However, I don't have any analytic approximation to their deflection and stresses under wire load. It is clear that the deflections will be larger than for a plate supported at both edges, and I find this option less desirable.

See note TNDC-96-25 for FEA calculations of this case.