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Feb. 28, 1996
BaBar TNDC-96-22

Endplates Under Pure Tension or Compression

Abstract

In the pursuit of a very thin endplate one is led to consider the limit where the endplate in under pure tension due to the wire load; no bending. The plate is then a sort of circularly symmetric suspension bridge. For plates with uniform axial load the resulting shape is a cubic. Should one desire a domelike endplate under pure compression, simply reverse the sign of the equation for $z(r)$.

1 Basic Equations

We consider an endplate that is symmetric about the z axis with an axial load, $P_z(r) =$ force/(area $\perp$ to the z axis). Let $T(r)$ be the total tension across the endplate at radius r. The endplate has shape $z(r)$ that is to be determined.

Assuming the load is entirely balanced by the tension in the endplate the equations for static equilibrium are that the radial component of the tension is constant,

$$\frac{T(r)}{\sqrt{1+z'^2}} = T_0 = \text{ constant}, \qquad \text{(radial)}$$

and that the difference in the vertical components of the tension on either side of a ring element of radial extent dr support the vertical load,

$$\left.\frac{Tz'}{\sqrt{1+z'^2}}\right|_{r+dr} - \left.\frac{Tz'}{\sqrt{1+z'^2}}\right|_{r} = dF_z = 2\pi P_z r dr. \qquad \text{(axial)}$$

On inserting the radial equation in the axial one we obtain the differential equation

$$z'' = 2\pi \frac{P_z(r)}{T_0} r.$$

One could also seek a pure-compression endplate which would be a domelike object. Simply reverse the sign of the tension above, leading to a sign change in the (linear) differential equation, and hence a sign change in the equation for the shape of the endplate found below.

In drift chambers the radial dependence of the axial load depends on the wire pattern. For a jet chamber the number of wires is the same at all radii, so for a chamber with inner radius r_1 and outer radius r_2 with total wire load F_0 we have

$$P_z(r) = \frac{F_0}{2\pi r(r_2 - r_1)}, \qquad z'' = \frac{F_0}{T_0(r_2 - r_1)},$$

$$z' = \frac{F_0 r}{T_0(r_2 - r_1)} + b, \qquad z = \frac{F_0 r^2}{2T_0(r_2 - r_1)} + br + c. \qquad \text{(jet chamber)}$$

The desired shape for a jet-chamber endplate is a parabola (as for a suspension bridge). For a chamber such as that of BaBar where the wire density is uniform we have

$$P_z(r) = \frac{F_0}{\pi(r_2^2 - r_1^2)}, \qquad z'' = \frac{2F_0 r}{T_0(r_2^2 - r_1^2)},$$

$$z' = \frac{F_0 r^2}{T_0(r_2^2 - r_1^2)} + b, \qquad z = ar^3 + br + c \qquad \text{with} \qquad a = \frac{F_0}{3T_0(r_2^2 - r_1^2)}. \qquad \text{(uniform chamber)}$$

Thus the shape of an endplate under pure tension from a uniform axial load is a cubic.

2 Boundary Conditions

The solution for the shape of the pure-tension endplate has three constants, T_0, b, and c, that must be determined from appropriate boundary conditions. We will complete the solution only for the case of uniform axial loading.

From considerations of the construction of a drift chamber, it is useful to choose one boundary condition as a statement of the fraction ϵ of the total axial load F_0 that is to be carried by the support at the inner radius r_1. Thus the axial load F_1 on the inner support is

$$F_1 = \epsilon F_0.$$

The constant ϵ can take on any value; while $0 \leq \epsilon \leq 1$ would be the more 'normal' range, $\epsilon < 0$ corresponds to the case that the inner support pulls on the endplate in the same direction as the wire load, and $\epsilon > 1$ occurs when the outer support pulls on the endplate.

The axial force F_1 on the inner support must balance the axial component of the tension in the endplate:

$$F_1 = - \left. \frac{Tz'}{\sqrt{1 + z'^2}} \right|_{r_1} = -T_0 z'(r_1) = -\frac{F_0 r_1^2}{r_2^2 - r_1^2} - T_0 b,$$

using the previous expresion for $z'(r)$, and hence

$$T_0 b = -\left(\frac{r_1^2}{r_2^2 - r_1^2} + \epsilon \right) F_0.$$

The remaining two boundary conditions are obtained by fixing the position and slope of the endplate at the outer radius r_2. [Having choosen the load F_1 on the inner support, one is not free in general to fix both the position and slope at the inner support, or even both the positions at the inner and outer support.] Without loss of generality we take

$$z(r_2) = 0, \qquad \text{and} \qquad z'(r_2) = z'_2$$

as the remaining two boundary conditions.

3 Form of the Endplate

Note that $T_0 z_2'$ is the axial force on the outer support, which is also $(1-\epsilon)F_0$ according to the condition on the force at the inner support. Hence

$$T_0 = \frac{(1-\epsilon)F_0}{z_2'}, \qquad \text{and so} \qquad a = \frac{z_2'}{3(1-\epsilon)(r_2^2 - r_1^2)}.$$

The expression for $T_0 b$ then yields

$$b = -\frac{z_2'[\epsilon r_2^2 + (1-\epsilon)r_1^2]}{(1-\epsilon)(r_2^2 - r_1^2)},$$

and the condition $z_2 = 0$ leads to

$$c = \frac{z_2' r_2[(\epsilon - 1/3)r_2^2 + (1-\epsilon)r_1^2]}{(1-\epsilon)(r_2^2 - r_1^2)}.$$

The cubic equation for the pure-tension endplate is then

$$z(r) = -\frac{z_2'[(r_2^3 - r^3)/3 - (r_2 - r)(\epsilon r_2^2 + (1-\epsilon)r_1^2)]}{(1-\epsilon)(r_2^2 - r_1^2)}.$$

The slope of the endplate is

$$z'(r) = \frac{z_2'[r^2 - (\epsilon r_2^2 + (1-\epsilon)r_1^2)]}{(1-\epsilon)(r_2^2 - r_1^2)},$$

which vanishes at

$$r_0 = \sqrt{\epsilon r_2^2 + (1-\epsilon)r_1^2}.$$

4 Example for the BaBar Drift Chamber

Table 1 gives an example relevant to the BaBar drift chamber, choosing $\epsilon = 0.4$ as the fraction of the load on the inner support tube, $z_2 = 0$, and slope $z_2' = 1.0$ at the outer support tube.

The pure-tension cubic is shown in Fig. 1 along with a circle of the same sagitta. The circle has radius 0.38 m and is centered at $(r, z) = (0.52, 0.26)$ m.

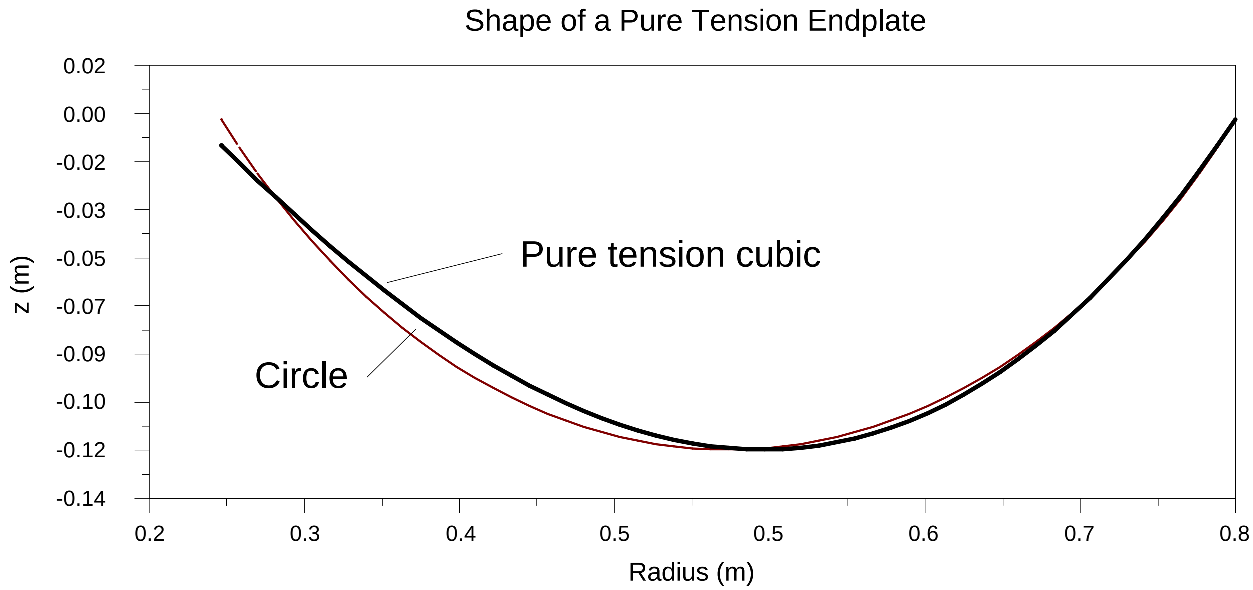

Figure 1: The pure-tension cubic form for the endplate with parameters given in Table 1. A circle with the same sagitta is shown for comparison.

Table 1: Sample parameters for a pure-tension endplate for BaBar.

Inner radius	0.24 m
Outer radius	0.80 m
ϵ	0.4
z'_2	1.0
z'_1	-0.667
a	0.954
b	-0.832
c	0.177
r_0	0.539 m
z_{min}	-0.122 m