Princeton/BaBar/TNDC-96-48
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Analytic Stress Analysis of Endplates Coupled by Inner and Outer Cylinders

1 Introduction

It has been pointed out that when a stepped front endplate is coupled to a flat rear endplate by inner and outer cylinders of the same length the endplates will distort until the loads are the same at both ends of the cylinders. We present a modification to the analytic stress analysis described in note Princeton/BaBar/96-41 that includes this effect. Results are presented for three scenarios:

A. Stepped front plate with equal-length cylinders.
B. Stepped front plate with unequal-length cylinders.
C. Flat front plate with equal-length cylinders.

2 Method

2.1 Formalism for a Single Endplate

*(Reproduced from Princeton/*BaBar*/96-41.)*

In the analytic method of stress analysis presented in Table 24 of *Roark* the mechanical state of the plate at some radius r is described by four variables that we write as components of a vector $\vec{V}$:

$V_{,1} = y =$ deflection of the plate;
$V_{,2} = \theta =$ angle of the plate;
$V_{,3} = M_r =$ radial bending moment per unit length of circumference;
$V_{,4} = Q =$ shear force per unit length of circumference.

Other quantities of interest can be deduced from these four. Thus the tangential bending moment per unit length of circumference is

$$M_t = \frac{\theta D(1-\nu^2)}{r} + \nu M_r,$$

where ν is Poisson's ratio (taken as 0.3 for aluminum) and D is the plate constant

$$D = \frac{Et^3}{12(1-\nu^2)},$$

where E is the Young's modulus (taken as 7×10^{10} Pa for aluminum) and t is the thickness of the plate (in the region of interest). The radial and tangential stresses are then related by

$$\sigma_r = \frac{6M_r}{t^2}, \qquad \text{and} \qquad \sigma_t = \frac{6M_r}{t^2}.$$

The values $\vec{V}(r)$ are related to the values $\vec{V}(b)$ at the inner radius b of an annular region by the matrix transformation

$$\vec{V}(r) = \overleftrightarrow{M}(r,b) \cdot \vec{V}(b) + \vec{C}(r,b),$$

where the matrix $\overleftrightarrow{M}$ and vector $\vec{C}$ are given in terms of analytic functions listed on pp. 398-399 and p. 404 of *Roark*.

We now consider a plate divided into n annular regions with inner radii $b_i,\ i = 1, ..., n$. We desire the transformation

$$\vec{V}(r) = \overleftrightarrow{M}(r,b_1) \cdot \vec{V}(b_1) + \vec{C}(r,b_1)$$

between properties at radius r and those at the innermost radius b_1. This can be built iteratively out of transformations valid only within region i,

$$\vec{V}(b_{i+1}) = \overleftrightarrow{M}_i(b_{i+1},b_i) \cdot \vec{V}(b_i) + \vec{C}_i(b_{i+1},b_i),$$

according to

$$\vec{C}(b_{i+1},b_1) = \overleftrightarrow{M}_i(b_{i+1},b_i) \cdot \vec{C}(b_i,b_1) + \vec{C}_i(b_{i+1},b_i),$$

$$\overleftrightarrow{M}(b_{i+1},b_1) = \overleftrightarrow{M}_i(b_{i+1},b_i) \cdot \overleftrightarrow{M}(b_i,b_1).$$

To complete a solution, we must also have the values $\vec{V}(b_1)$ at the inner radius. However, these are typically not completely known. Rather some mixtures of values at the inner and outer radius of the plate are specified by the boundary conditions.

2.2 Coupling of Two Endplates by Cylinders

We now consider the case of two endplates, labelled F for front and R for rear. Correspondingly we consider the variable vectors $\vec{V}_F$ and $\vec{V}_R$, transformation matrices $\overleftrightarrow{M}_F$ and $\overleftrightarrow{M}_R$ and constant vectors $\vec{C}_R$ and $\vec{C}_R$.

To complete the solution here we need the 8 values of the variable vectors at the inner radius $b_1 = b$.

We are free to fix one point on the chamber, which I take to be the inner radius of the front endplate:

$$y_F(b) = V_{F,1}(b) = 0.$$

2.2.1 Bending Moments at the Inner and Outer Radii

If we suppose the joints at the inner and outer radii of the endplates provide only a simple support, then the bending moments must all be zero there:

$$V_{F,3}(a) = V_{F,3}(b) = V_{R,3}(a) = V_{R,3}(b) = 0.$$

However, it proves to be straightforward to allow these four bending moments to have specified values and the formalism below includes this option.

2.2.2 Forces on the Inner and Outer Cylinders

The condition whose significance was not fully appreciated before is that the shear forces (per unit length of circumference) must be the same for both endplates at the outer radius since these are coupled by the outer cylinder:

$$V_{F,4}(a) = V_{R,4}(a).$$

Likewise the shear forces must be equal for both endplates at the inner radius:

$$V_{F,4}(b) = V_{R,4}(b).$$

2.2.3 Compression of the Cylinders

Additional constraints arise from the lengths of the inner and outer cylinders. These are not constant, but undergo compression due to the wire load. If a cylinder made of a material with Young's modulus E has length l and thickness t then its compression Δy under a load Q per unit length of circumference is

$$\Delta y = \frac{Ql}{tE}.$$

In the case of the inner cylinder which is made of aluminum of length l_{Al} and thickness t_{Al} and beryllium of length l_{Be} and thickness t_{Be} the compression is

$$\Delta y = Q\left(\frac{l_{\mathrm{Al}}}{t_{\mathrm{Al}}E_{\mathrm{Al}}} + \frac{l_{\mathrm{Be}}}{t_{\mathrm{Be}}E_{\mathrm{Be}}}\right).$$

I summarize these relations in the form

$$\Delta y = QS,$$

and suppose the numerical values S_a and S_b for the outer and inner cylinders are known.

To incorporate these facts we must pay attention to some sign conventions. Endplate displacements $y = V_{,1}$ are positive when away from the direction of the wire load. That is, the y_F and y_R axes have opposite directions! Further, I take $y_R = 0$ to be the position of the inner radius of the rear endplate under no wire load. Thus the y_F and y_R axes have different origins.

Also, a shear force Q is considered positive if the material at the smaller radius exerts a force in the $+y$ direction on the material at the larger radius. In particular, $Q(b) = V_{,4}(b) > 0$ but $Q(a) < 0$ for our chamber.

Then the condition on the length of the inner cylinder under load becomes

$$\Delta y(b) = V_{R,1}(b) = -S_b Q(b) = -S_b V_{F,4}(b),$$

recalling that $V_{F,1}(b) = 0$ by definition.

We allow the outer cylinder to be longer than the inner cylinder by an amount Δl under no load. Then the compression of the outer cylinder under load is related by

$$\Delta y(a) = V_{R,1}(a) + V_{F,1}(a) - \Delta l = S_a V_{F,4}(a),$$

noting the sign conventions, *etc.*

2.2.4 A Relation Derived from $V_{R,1}(a)$

We now deduce various relations among the values $\vec{V}$ at the inner and outer radii using the known transformations $\overleftrightarrow{M}(a, b)$ and constants $\vec{C}(a, b)$. We will find three equation involving the three unknowns $V_{F,2}(b)$, $V_{F,4}(b)$ and $V_{R,2}(b)$.

First we note that $V_{F,1}(b) = 0$ so that

$$V_{F,1}(a) = M_{F,1,2}V_{F,2}(b) + M_{F,1,3}V_{F,3}(b) + M_{F,1,4}V_{F,4}(b) + C_{F,1},$$

and

$$V_{F,4}(a) = M_{F,4,2}V_{F,2}(b) + M_{F,4,3}V_{F,3}(b) + M_{F,4,4}V_{F,4}(b) + C_{F,4},$$

which can be combined with the expression for $\Delta y(a)$ to give

$$\begin{aligned} V_{R,1}(a) = \ & (-M_{F,1,2} + S_a M_{F,4,2})V_{F,2}(b) + (-M_{F,1,3} + S_a M_{F,4,3})V_{F,3}(b) \\ & +(-M_{F,1,4} + S_a M_{F,4,4})V_{F,4}(b) - C_{F,1} + S_a C_{F,4} + \Delta l. \end{aligned}$$

We also have the direct transformation

$$V_{R,1}(a) = M_{R,1,1}V_{R,1}(b) + M_{R,1,2}V_{R,2}(b) + M_{R,1,3}V_{R,3}(b) + M_{R,1,4}V_{R,4}(b) + C_{R,1}.$$

But $V_{R,1}(b)$ is also known from the relation on $\Delta y(b)$ and $V_{R,4}(b) = V_{F,4}(b)$, so all the above combine to yield

$$\begin{aligned} &(S_a M_{F,4,2} - M_{F,1,2})V_{F,2}(b) + (S_a M_{F,4,4} + S_b M_{R,1,1} - M_{F,1,4} - M_{R,1,4})V_{F,4}(b) - M_{R,1,2}V_{R,2}(b) \\ &= C_{F,1} + C_{R,1} - S_a C_{F,4} - \Delta l + M_{R,1,3}V_{R,3}(b) + (M_{F,1,3} - S_a M_{F,4,3})V_{F,3}(b). \end{aligned}$$

2.2.5 From $V_{F,3}(a)$

A second relation derives from the transformation leading to $V_{F,3}(a)$:

$$M_{F,3,2}V_{F,2}(b) + M_{F,3,4}V_{F,4}(b) = -C_{F,3} - M_{F,3,3}V_{F,3}(b) + V_{F,3}(a).$$

2.2.6 From $V_{R,3}(a)$

The third relation derives from the transformation leading to $V_{R,3}(a)$:

$$(M_{R,3,4} - S_b M_{R,3,1})V_{F,4}(b) + M_{R,3,2}V_{R,2}(b) = -C_{R,3} - M_{R,3,3}V_{R,3}(b) + V_{R,3}(a).$$

2.2.7 Completing the Solution

The three unknowns $V_{F,2}(b)$, $V_{F,4}(b)$ and $V_{R,2}(b)$ are now determined by the solution of these three simultaneous linear equations. The remaining values of $\vec{V}_R(b)$ follow at once from relations stated above.

The FORTRAN program `case2_chamber.for` that implements this procedure can be found in the Princeton Technical Notes Web page.

2.2.8 Case of Known Angles at the Inner and Outer Radii

Suppose instead of having known moments $V_{,3}$ at the inner and outer radii, the angles $V_{,2}$ are known there. The preceding formalism holds with only slight modifications.

The constraints discussed in secs. 2.2.2 and 2.2.3 still hold. It proves convenient to take $V_{F,3}(b)$, $V_{F,4}(b)$ and $V_{R,3}(b)$ as the three unknown parameters. Then we can immediately rewrite the result of sec. 2.2.4 as

$$\begin{gathered}(S_a M_{F,4,3} - M_{F,1,3})V_{F,3}(b) + (S_a M_{F,4,4} + S_b M_{R,1,1} - M_{F,1,4} - M_{R,1,4})V_{F,4}(b) - M_{R,1,3}V_{R,3}(b) \\ = C_{F,1} + C_{R,1} - S_a C_{F,4} - \Delta l + M_{R,1,2}V_{R,2}(b) + (M_{F,1,2} - S_a M_{F,4,2})V_{F,2}(b).\end{gathered}$$

A second relation derives from the transformation leading to $V_{F,2}(a)$:

$$M_{F,2,3}V_{F,3}(b) + M_{F,2,4}V_{F,4}(b) = -C_{F,2} - M_{F,2,2}V_{F,2}(b) + V_{F,2}(a).$$

The third relation derives from the transformation leading to $V_{R,3}(a)$:

$$(M_{R,2,4} - S_b M_{R,2,1})V_{F,4}(b) + M_{R,2,3}V_{R,3}(b) = -C_{R,2} - M_{R,2,2}V_{R,2}(b) + V_{R,2}(a).$$

The rest of the solution proceeds in a manner similar to the case of known moments.

3 Corrections for Holes

*(Reproduced from Princeton/*BaBar*/96-41.)*

The analytic calculation described above assumes the plate is homogeneous within each annular region. In particular, there are no holes. To a good approximation the effect of small holes drilled in the plate is twofold:

1. The modulus is reduced. Both our ALGOR finite-element analysis and John Hodgson's ANSYS calculation (June 27, 1996) indicate that the effective modulus is 1/1.5 times the nominal 7×10^{10} Pa for aluminum.

2. The peak stress around the holes is greater than the stress in a plate with no holes. Table 37, Cases 6 and 7 of *Roark* discusses analytic calculations of the stress-concentration factor around holes for various idealized patterns of bending; this factor varies between 2 and 3. Our ALGOR finite-element analysis indicates a factor of 2.2 for the geometry of the BaBar endplate, while Hodgson's result indicate that a factor of 4.1 holds. This discrepancy is not understood. The present version of the program `case2_step.for` uses a value of 4.1.

4 Results Assuming Simply Supported Endplates

If the joints between the endplates and the cylinders transmit no bending moments we have the case of simply supported endplates. This corresponds to setting

$$V_{F,3}(a) = V_{F,3}(b) = V_{R,3}(a) = V_{R,3}(b) = 0.$$

in the formalism of sec. 2.2.

4.1 Stepped Front Plate with Equal-Length Cylinders

Many of the parameters describing the 51 regions used in the stress analysis are listed in Table 1 for a front endplate stepped between 24 and 12 mm. The wire tensions are taken as 34, 86 and 182 gm for the 20-, 80- and 120-μm wires, respectively, following TNDC-96-43 by C. Hearty. The total wire load is 3502 kg for the 7104 sense wires, 7104 clearing wires and 14560 field wires. The endplate simulated in Table 1 has thickness $t = 24$ mm out to a radius of 47 cm, and thickness 12 mm at larger radii. Tables 2 and 3 summarize some of the results of the stress analysis based on the parameters listed in Table 1.

Figure 1 presents some of the key results in graphical form. Of particular note is the displacements of the endplates at their outer radii: the front endplate moves about 1 mm away from the interior while the rear endplate moves 1 mm towards the interior.

The chamber takes on a mild parallelogram shape in its half section. This occurs because the thin, outer region of the front plate is not as stiff as the rear plate. It must bend a lot to develop enough force to resist the push of the outer cylinder.

The 1-mm-displacement of the outer radius of the rear chamber has been considered so large as to be unacceptable for mounting of the electronics boards.

4.2 Stepped Front Plate with Unequal-Length Cylinders

The most favorable scenario for mounting the electronics on the rear endplate is that the displacements at the inner and outer radius be equal. [They cannot be zero due to compression of the inner and outer cylinders.]

If the outer cylinder is made longer than the inner cylinder the front plate will be forced into greater distortion, causing it to develop greater restoring forces that can push the rear endplate back into the desired state. Calculations show that an extra length of 3.1 mm is required for this. See Fig. 2.

Table 1: Input parameters for a stress analysis of a stepped endplate.

Region	Inner Radius (cm)	No. of Sense Wires	No. of Clear Wires	No. of Field Wires	Load Pressure (Pa)	Modulus Factor	Hole Stress Factor	t (mm)	Plate Constant (N)
1	23.5	0	0	0	0.0	1.00	1.0	24.	90495.
2	25.3	96	192	288	32.9	0.67	4.1	24.	60330.
3	26.6	96	0	192	18.4	0.67	4.1	24.	60330.
4	27.8	96	0	192	17.8	0.67	4.1	24.	60330.
5	29.0	96	192	192	24.7	0.67	4.1	24.	60330.
6	30.2	0	0	0	0.0	1.00	1.0	24.	90495.
7	31.2	112	224	224	26.3	0.67	4.1	24.	60330.
8	32.4	112	0	224	17.8	0.67	4.1	24.	60330.
9	33.6	112	0	224	17.2	0.67	4.1	24.	60330.
10	34.8	112	224	224	24.1	0.67	4.1	24.	60330.
11	36.0	0	0	0	0.0	1.00	1.0	24.	90495.
12	36.5	128	256	256	26.0	0.67	4.1	24.	60330.
13	37.6	128	0	256	17.6	0.67	4.1	24.	60330.
14	38.8	128	0	256	17.1	0.67	4.1	24.	60330.
15	40.0	128	256	256	23.8	0.67	4.1	24.	60330.
16	41.2	0	0	0	0.0	1.00	1.0	24.	90495.
17	41.7	144	288	288	25.7	0.67	4.1	24.	60330.
18	42.9	144	0	288	17.4	0.67	4.1	24.	60330.
19	44.0	144	0	288	17.0	0.67	4.1	24.	60330.
20	45.2	144	288	288	23.9	0.67	4.1	24.	60330.
21	46.4	0	0	0	0.0	1.00	1.0	24.	90495.
22	47.5	176	352	352	27.1	0.67	4.1	12.	7541.
23	48.7	176	0	352	19.0	0.67	4.1	12.	7541.
24	49.8	176	0	352	18.5	0.67	4.1	12.	7541.
25	51.0	176	352	352	25.7	0.67	4.1	12.	7541.
26	52.2	0	0	0	0.0	1.00	1.0	12.	11312.
27	52.7	192	384	384	27.2	0.67	4.1	12.	7541.
28	53.9	192	0	384	18.5	0.67	4.1	12.	7541.
29	55.1	192	0	384	18.2	0.67	4.1	12.	7541.
30	56.2	192	384	384	25.5	0.67	4.1	12.	7541.
31	57.4	0	0	0	0.0	1.00	1.0	12.	11312.
32	57.9	208	416	416	26.8	0.67	4.1	12.	7541.
33	59.1	208	0	416	18.3	0.67	4.1	12.	7541.
34	60.3	208	0	416	18.0	0.67	4.1	12.	7541.
35	61.5	208	416	416	25.5	0.67	4.1	12.	7541.
36	62.6	0	0	0	0.0	1.00	1.0	12.	11312.
37	63.7	224	448	448	26.0	0.67	4.1	12.	7541.
38	64.9	224	0	448	18.2	0.67	4.1	12.	7541.
39	66.1	224	0	448	17.7	0.67	4.1	12.	7541.
40	67.2	224	448	448	24.9	0.67	4.1	12.	7541.
41	68.4	0	0	0	0.0	1.00	1.0	12.	11312.
42	68.9	240	480	480	26.0	0.67	4.1	12.	7541.
43	70.1	240	0	480	17.9	0.67	4.1	12.	7541.
44	71.3	240	0	480	17.6	0.67	4.1	12.	7541.
45	72.5	240	480	480	24.8	0.67	4.1	12.	7541.
46	73.6	0	0	0	0.0	1.00	1.0	12.	11312.
47	74.1	256	512	512	25.8	0.67	4.1	12.	7541.
48	75.3	256	0	512	17.7	0.67	4.1	12.	7541.
49	76.5	256	0	512	17.5	0.67	4.1	12.	7541.
50	77.7	256	512	768	29.0	0.67	4.1	12.	7541.
51	79.0	0	0	0	0.0	1.00	1.0	12.	11312.

Table 2: Deflections and stress for a 24/12-mm-thick front endplate.

Region	Mid Radius (cm)	Δz (mm)	θ (mrad)	M_r (kN)	M_t (kN)	Load (kg)	σ_r (MPa)	σ_t (MPa)
1	24.4	-0.052	-5.7	19.0	-1892.4	1518.	0.2	-19.7
2	26.0	-0.141	-5.6	71.2	-1140.3	1501.	3.0	-48.7
3	27.2	-0.211	-5.5	126.1	-1047.3	1452.	5.4	-44.7
4	28.4	-0.275	-5.4	174.2	-965.9	1416.	7.4	-41.3
5	29.6	-0.338	-5.3	218.7	-890.5	1372.	9.3	-38.0
6	30.7	-0.397	-5.2	250.0	-1282.0	1348.	2.6	-13.4
7	31.8	-0.456	-5.1	280.9	-767.0	1316.	12.0	-32.8
8	33.0	-0.515	-5.0	317.8	-704.5	1263.	13.6	-30.1
9	34.2	-0.573	-4.8	351.6	-646.2	1220.	15.0	-27.6
10	35.4	-0.629	-4.7	382.4	-591.8	1168.	16.3	-25.3
11	36.2	-0.668	-4.6	399.9	-901.2	1148.	4.2	-9.4
12	37.1	-0.707	-4.6	416.7	-523.2	1101.	17.8	-22.3
13	38.2	-0.760	-4.4	440.7	-476.6	1041.	18.8	-20.4
14	39.4	-0.811	-4.3	462.1	-432.9	991.	19.7	-18.5
15	40.6	-0.861	-4.2	481.0	-391.9	931.	20.5	-16.7
16	41.4	-0.896	-4.1	491.1	-629.9	902.	5.1	-6.6
17	42.3	-0.930	-4.0	500.4	-341.0	855.	21.4	-14.6
18	43.5	-0.976	-3.8	513.2	-306.4	786.	21.9	-13.1
19	44.6	-1.020	-3.7	523.6	-274.2	730.	22.4	-11.7
20	45.8	-1.063	-3.6	531.9	-244.2	662.	22.7	-10.4
21	46.9	-1.103	-3.5	535.2	-418.7	622.	5.6	-4.4
22	48.1	-1.141	-3.0	542.3	137.3	572.	92.6	23.5
23	49.3	-1.171	-2.1	552.8	153.8	488.	94.4	26.3
24	50.4	-1.190	-1.2	560.3	168.6	419.	95.7	28.8
25	51.6	-1.199	-0.3	565.2	182.1	335.	96.6	31.1
26	52.4	-1.199	0.2	566.8	191.5	287.	23.6	8.0
27	53.3	-1.195	0.8	567.5	197.3	231.	96.9	33.7
28	54.5	-1.181	1.7	565.8	207.4	139.	96.7	35.4
29	55.6	-1.156	2.5	561.7	216.1	64.	96.0	36.9
30	56.8	-1.121	3.4	555.3	223.4	-28.	94.9	38.2
31	57.7	-1.090	3.9	549.1	250.0	-83.	22.9	10.4
32	58.5	-1.055	4.5	542.3	230.2	-141.	92.6	39.3
33	59.7	-0.997	5.3	530.1	234.2	-241.	90.6	40.0
34	60.9	-0.930	6.1	515.7	237.0	-323.	88.1	40.5
35	62.0	-0.855	6.8	499.5	238.5	-423.	85.3	40.7
36	63.2	-0.775	7.4	482.1	276.8	-482.	20.1	11.5
37	64.3	-0.688	7.9	464.2	236.1	-545.	79.3	40.3
38	65.5	-0.591	8.6	443.0	234.3	-652.	75.7	40.0
39	66.7	-0.486	9.2	420.0	231.4	-741.	71.7	39.5
40	67.8	-0.374	9.8	395.0	227.4	-848.	67.5	38.8
41	68.7	-0.290	10.1	376.0	272.9	-917.	15.7	11.4
42	69.5	-0.204	10.5	356.5	218.8	-980.	60.9	37.4
43	70.7	-0.077	10.9	326.8	211.9	-1095.	55.8	36.2
44	71.9	0.054	11.4	295.4	203.7	-1190.	50.5	34.8
45	73.0	0.191	11.7	262.2	194.5	-1305.	44.8	33.2
46	73.9	0.290	11.9	237.5	241.4	-1380.	9.9	10.1
47	74.7	0.391	12.1	212.3	179.1	-1446.	36.3	30.6
48	75.9	0.536	12.4	174.8	167.2	-1569.	29.9	28.6
49	77.1	0.683	12.5	135.6	154.1	-1670.	23.2	26.3
50	78.3	0.841	12.7	92.3	139.1	-1815.	15.8	23.8
51	80.0	1.045	12.7	33.8	171.2	-1904.	1.4	7.1

Table 3: Deflections and stress for a 24-mm-thick rear endplate.

Region	Mid Radius (cm)	Δz (mm)	θ (mrad)	M_r (kN)	M_t (kN)	Load (kg)	σ_r (MPa)	σ_t (MPa)
1	24.4	-0.128	-4.7	31.5	-1557.2	1518.	0.3	-16.2
2	26.0	-0.201	-4.6	100.0	-926.6	1501.	4.3	-39.6
3	27.2	-0.259	-4.5	163.1	-842.0	1452.	7.0	-36.0
4	28.4	-0.312	-4.4	218.1	-767.4	1416.	9.3	-32.8
5	29.6	-0.363	-4.3	268.7	-698.1	1372.	11.5	-29.8
6	30.7	-0.412	-4.2	306.6	-1010.0	1348.	3.2	-10.5
7	31.8	-0.459	-4.1	343.5	-583.9	1316.	14.7	-24.9
8	33.0	-0.507	-4.0	384.7	-525.6	1263.	16.4	-22.4
9	34.2	-0.554	-3.9	422.2	-471.1	1220.	18.0	-20.1
10	35.4	-0.599	-3.8	456.5	-420.0	1168.	19.5	-17.9
11	36.2	-0.630	-3.7	476.7	-659.6	1148.	5.0	-6.9
12	37.1	-0.660	-3.6	496.1	-355.7	1101.	21.2	-15.2
13	38.2	-0.702	-3.4	522.7	-311.8	1041.	22.3	-13.3
14	39.4	-0.741	-3.3	546.6	-270.5	991.	23.3	-11.6
15	40.6	-0.780	-3.2	567.7	-231.8	931.	24.2	-9.9
16	41.4	-0.806	-3.1	579.7	-406.6	902.	6.0	-4.2
17	42.3	-0.831	-3.0	590.8	-183.8	855.	25.2	-7.9
18	43.5	-0.865	-2.8	605.3	-151.0	786.	25.9	-6.4
19	44.6	-0.898	-2.7	617.4	-120.4	730.	26.4	-5.1
20	45.8	-0.929	-2.5	627.2	-92.0	662.	26.8	-3.9
21	46.9	-0.957	-2.4	632.5	-208.6	622.	6.6	-2.2
22	48.1	-0.984	-2.3	637.4	-47.9	572.	27.2	-2.0
23	49.3	-1.010	-2.2	641.1	-24.8	488.	27.4	-1.1
24	50.4	-1.035	-2.0	642.5	-3.8	419.	27.4	-0.2
25	51.6	-1.058	-1.9	641.7	15.4	335.	27.4	0.7
26	52.4	-1.073	-1.8	639.0	-65.1	287.	6.7	-0.7
27	53.3	-1.088	-1.7	635.6	37.7	231.	27.1	1.6
28	54.5	-1.107	-1.6	629.0	52.6	139.	26.9	2.2
29	55.6	-1.125	-1.4	620.3	65.9	64.	26.5	2.8
30	56.8	-1.141	-1.3	609.6	77.5	-28.	26.0	3.3
31	57.7	-1.152	-1.2	600.2	26.7	-83.	6.3	0.3
32	58.5	-1.162	-1.1	590.3	89.6	-141.	25.2	3.8
33	59.7	-1.175	-1.0	574.3	97.3	-241.	24.5	4.2
34	60.9	-1.186	-0.9	556.6	103.6	-323.	23.8	4.4
35	62.0	-1.196	-0.8	537.0	108.3	-423.	22.9	4.6
36	63.2	-1.204	-0.7	516.1	79.9	-482.	5.4	0.8
37	64.3	-1.212	-0.6	494.8	111.1	-545.	21.1	4.7
38	65.5	-1.219	-0.5	470.8	112.2	-652.	20.1	4.8
39	66.7	-1.224	-0.4	445.2	111.9	-741.	19.0	4.8
40	67.8	-1.229	-0.3	417.7	110.3	-848.	17.8	4.7
41	68.7	-1.232	-0.3	396.8	96.3	-917.	4.1	1.0
42	69.5	-1.234	-0.2	375.4	104.9	-980.	16.0	4.5
43	70.7	-1.236	-0.2	343.5	100.1	-1095.	14.7	4.3
44	71.9	-1.238	-0.1	310.0	94.1	-1190.	13.2	4.0
45	73.0	-1.239	-0.1	274.9	86.9	-1305.	11.7	3.7
46	73.9	-1.239	0.0	248.6	80.0	-1380.	2.6	0.8
47	74.7	-1.239	0.0	221.9	74.0	-1446.	9.5	3.2
48	75.9	-1.239	0.1	182.6	63.8	-1569.	7.8	2.7
49	77.1	-1.238	0.1	141.7	52.5	-1670.	6.1	2.2
50	78.3	-1.237	0.1	96.7	39.2	-1815.	4.1	1.7
51	80.0	-1.235	0.1	35.5	23.8	-1904.	0.4	0.2

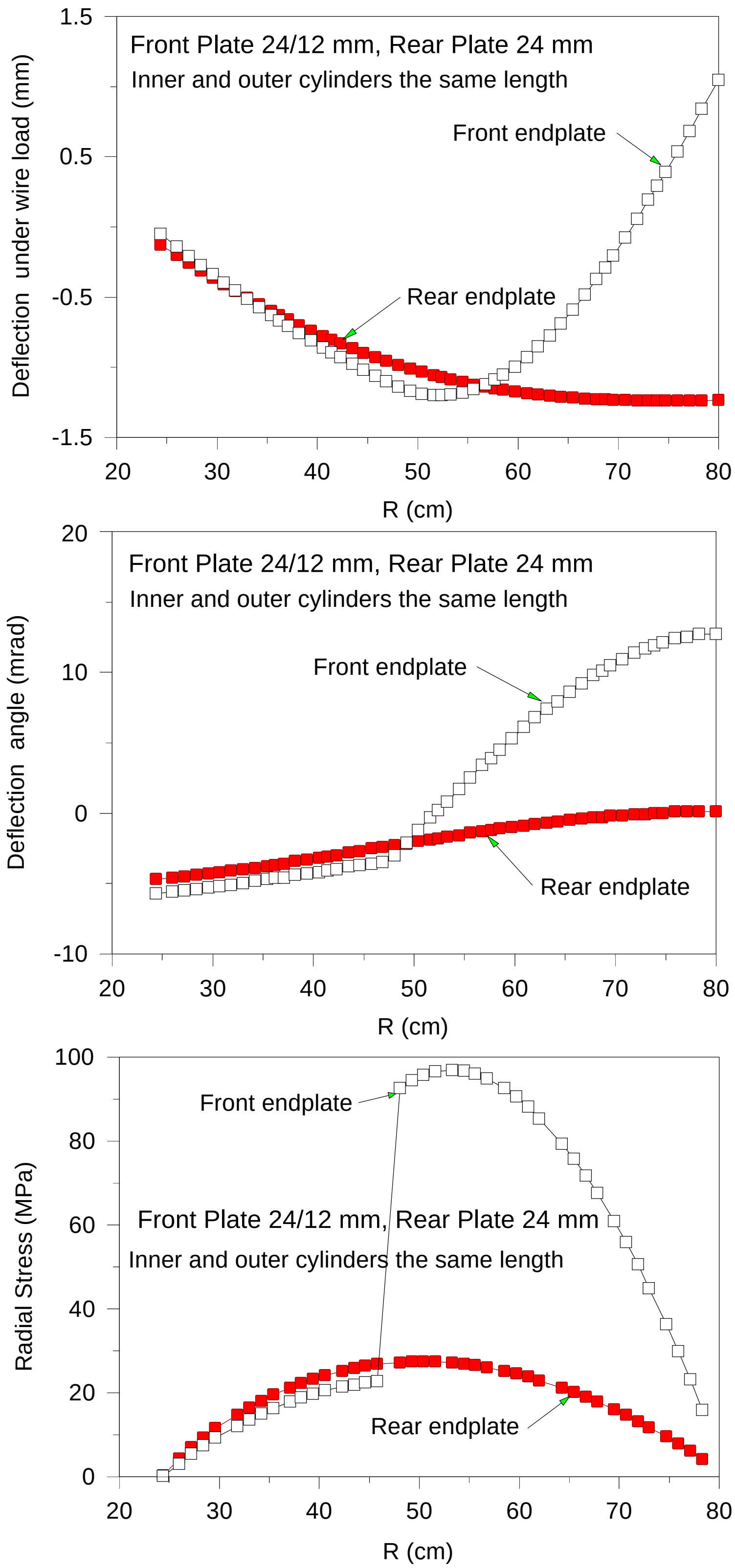

Figure 1: The displacement, deflection angle and radial stress in a 24/12-mm-thick front endplate coupled to a 24-mm-thick rear endplate by inner and outer cylinders of the same length.

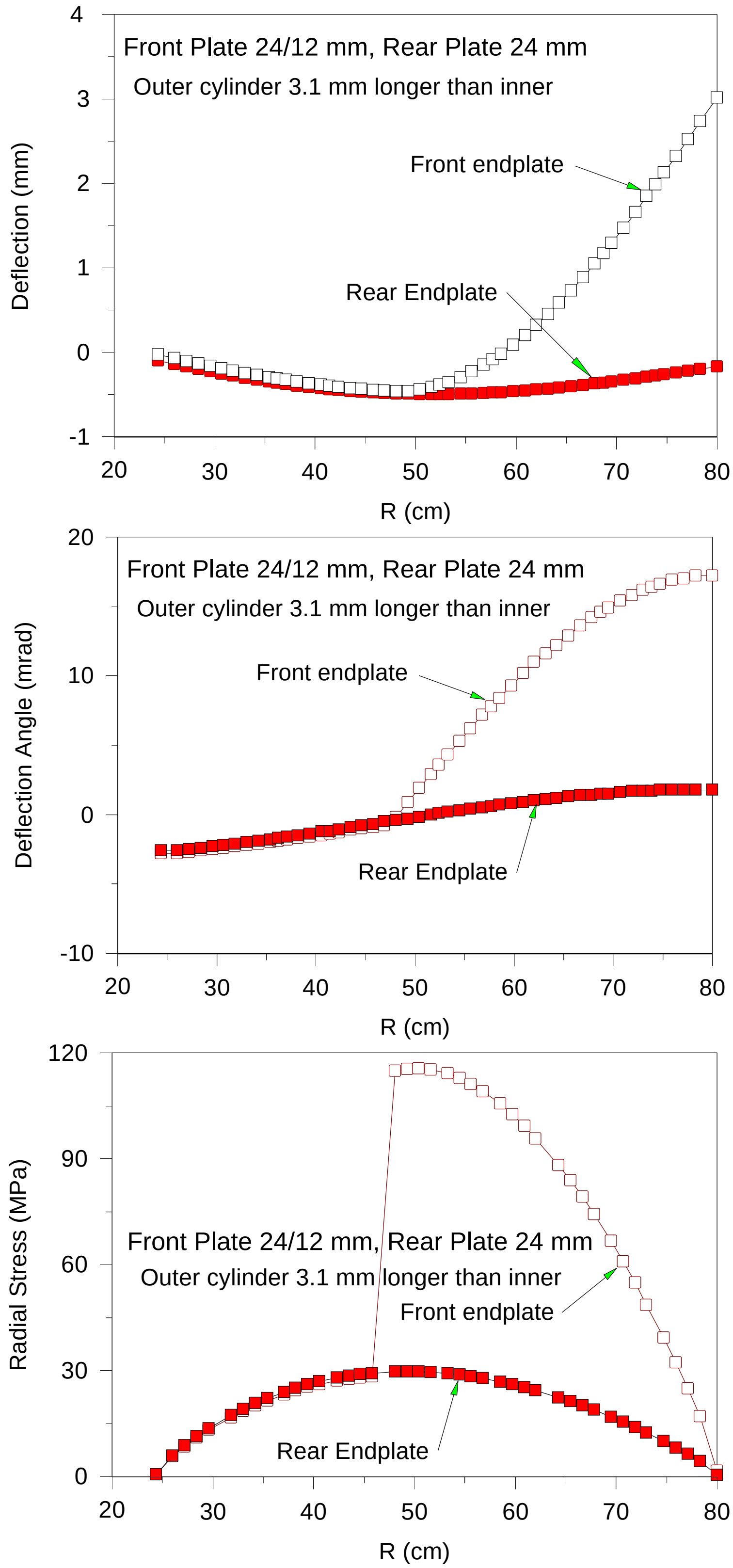

Figure 2: The displacement, deflection angle and radial stress in a 24/12-mm-thick front endplate coupled to a 24-mm-thick rear endplate by an outer cylinder 3.1 mm longer than the inner cylinder.

To prepare the chamber for assembly in such a configuration, the front endplate would have to be distorted by 3.1 mm between its outer and inner radii. This requires a force of 250 kg distributed around its circumference. Figure 3 shows the profile of the front endplate if only this force is applied. Note that the maximum difference between the plate so distorted and the final profile under wire load is only 1.6 mm – the same difference as would occur if the inner and outer cylinders had the same lengths.

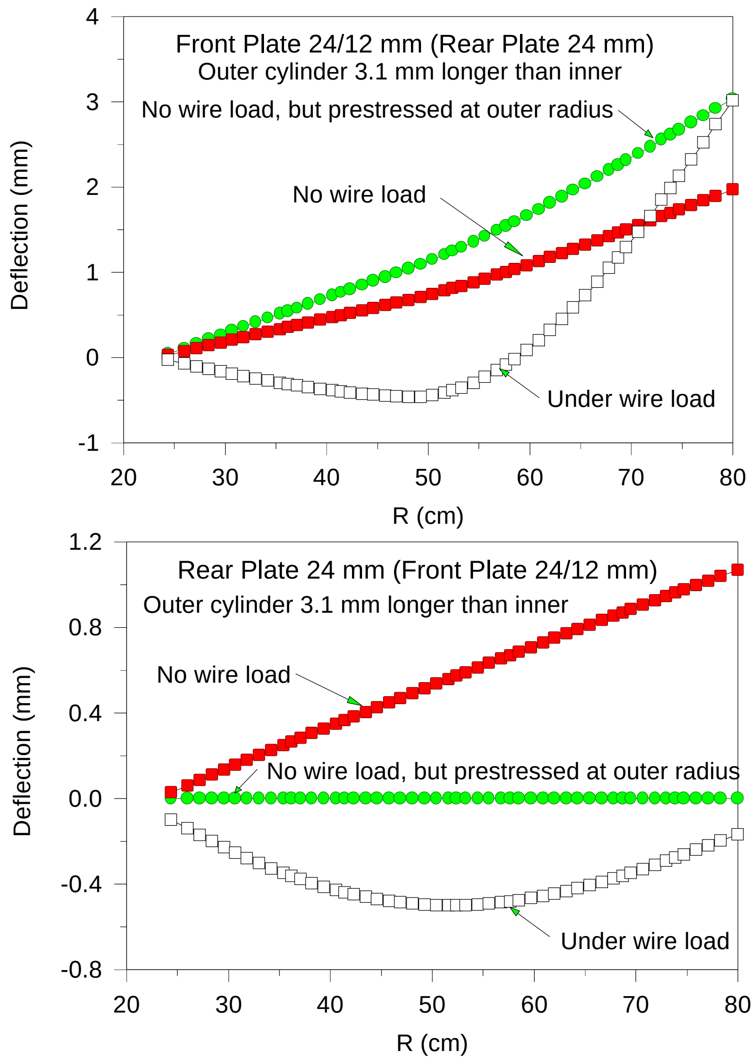

Figure 3: Displacements of a stepped front endplate and uniform rear endplate when coupled by an outer cylinder 3.1 mm longer than the inner cylinder for three possible stages of the chamber assembly.

Figure 3 also reveals that if the 3.1-mm-longer outer cylinder were installed together with the endplates prior to stringing the chamber, and the prestressing fixtures were released, the chamber would deform by about 1 mm, but this time the rear endplate would move away from the interior and the front endplate would moves towards it.

4.3 Flat Front Plate with Equal-Length Cylinders

A simpler way to keep the displacements of the rear endplate equal at the inner and outer radii is to have the front endplate flat. Then now matter what the thickness of the front plate, the load sharing between inner and outer radius is the same and there is not tendency for the chamber to distort into a parallelogram.

A thickness of 14 mm for the front plate would keep the deflection under wire load the same as for the stepped 24/12-mm plate. Figure 4 summarizes results of calculations for this case.

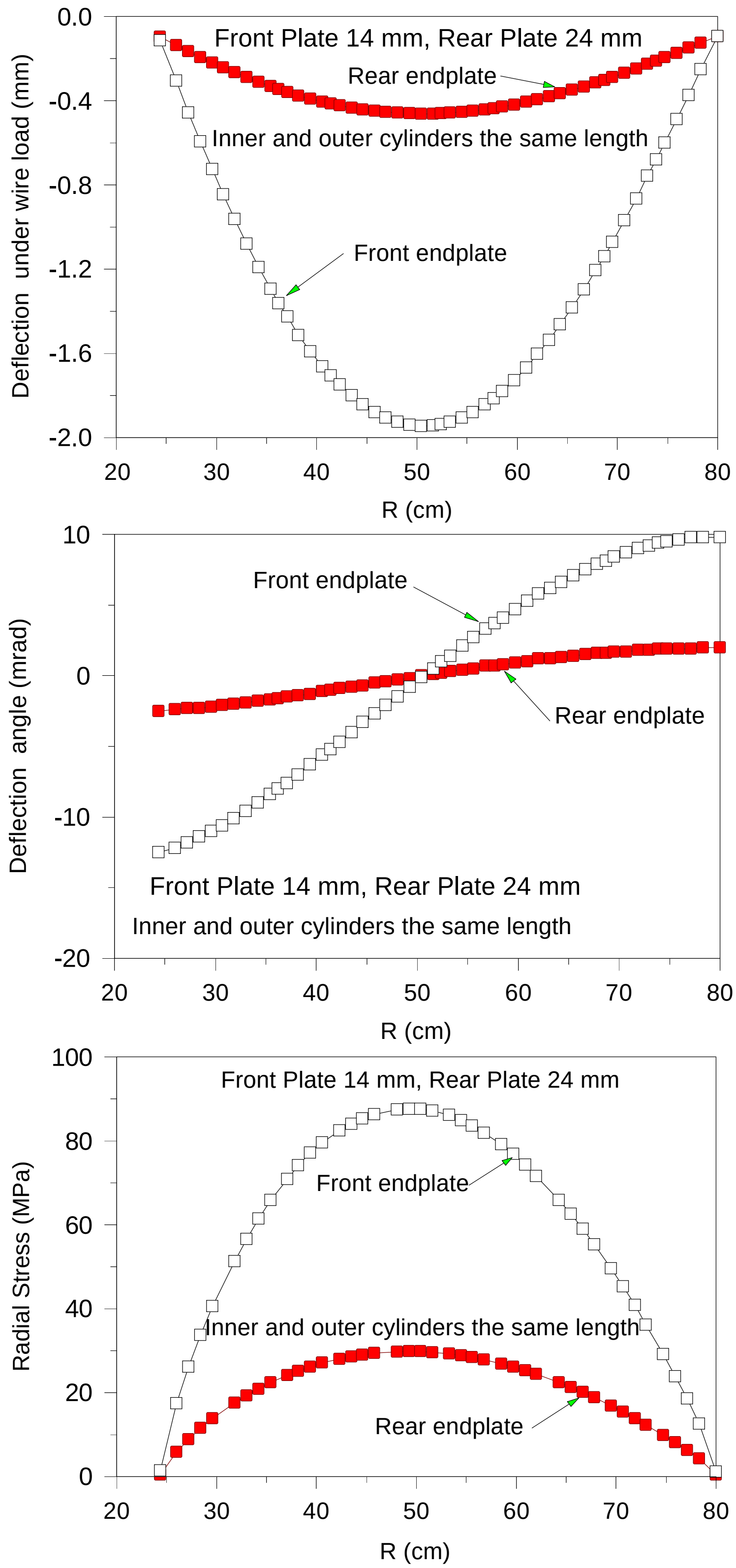

Figure 4: The displacement, deflection angle and radial stress in a 14-mm-thick front endplate coupled to a 24-mm-thick rear endplate by inner and outer cylinders of the same length.